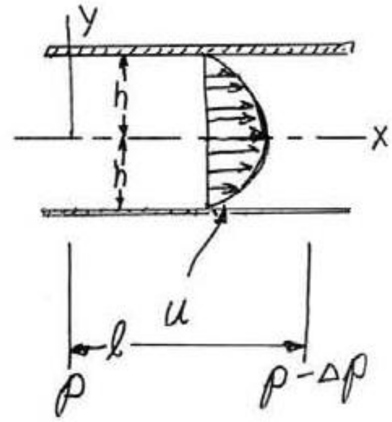
6.84

6.84 Oil ($\mu = 0.4$ N·s/m^2) flows between two fixed horizontal infinite parallel plates with a spacing of 5 mm. The flow is laminar and steady with a pressure gradient of -900 (N/m^2) per unit meter. Determine the volume flowrate per unit width and the shear stress on the upper plate.



From Eq. (6.136)

q = volume flowrate per unit width out of the paper

$$= \frac{2h^3 \Delta p}{3\mu \ell} \quad \text{where } \frac{\partial p}{\partial x} = -\frac{\Delta p}{\ell}$$

For this flow $2h = 5\,mm$ or $h = 2.5mm = 2.5 \times 10^{-3} m$

and $\Delta p/\ell = (+900\,N/m^2)/m = +900\,N/m^3$

Thus,

$$q = \frac{2(2.5\times10^{-3}m)^3(900\frac{N}{m^3})}{3\,(0.4N{\cdot}s/m^2)} = \underline{\underline{2.34\times10^{-5}\,\frac{m^2}{s}}}$$

The shear stress is $\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$

where

$$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2 - h^2) = -\frac{\Delta p}{2\mu\ell}(y^2 - h^2)$$

and

$v = 0$

Hence,

$$\tau_{xy} = -\frac{\Delta p}{2\mu\ell}(2y)\mu = -\frac{\Delta p}{\ell}y$$

On the upper plate $y = h$ so that

τ_{upper} = magnitude of shear stress on upper plate

$$= \frac{\Delta p}{\ell}h = (900\frac{N}{m^3})(2.5\times10^{-3}m) = \underline{\underline{2.25\,\frac{N}{m^2}}}$$ acting in the positive x-direction (the direction of flow).