6.16

6.16 The stream function for an incompressible, two-dimensional flow field is

$$\psi = ay - by^3$$

where a and b are constants. Is this an irrotational flow? Explain.

For the flow to be irrotational,

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad \text{(Eq. 6.12)}$$

and for the stream function given,

$$u = \frac{\partial \psi}{\partial y} = a - 3by^2$$

$$v = -\frac{\partial \psi}{\partial x} = 0$$

Thus,

$$\frac{\partial u}{\partial y} = -6by \qquad \frac{\partial v}{\partial x} = 0$$

so that

$$\omega_z = \frac{1}{2}\left[0 - (-6by)\right] = 3by$$

Since $\omega_z \neq 0$ flow is not irrotational (unless $b = 0$). NO.