5.112

5.112 A hydraulic turbine is provided with 4.25 m^3/s of water at 415 kPa. A vacuum gage in the turbine discharge 3 m below the turbine inlet centerline reads 250 mm Hg vacuum. If the turbine shaft output power is 1100 kW, calculate the power loss through the turbine. The supply and discharge pipe inside diameters are identically 80 mm.

We consider the turbine inlet and discharge to be sections (1) and (2).

For flow from sections (1) to (2) Eq. 5.82 yields

$$\text{loss} = \frac{P_1 - P_2}{\rho} + g(z_1 - z_2) - w_{\substack{\text{shaft} \\ \text{net out}}} \qquad (1)$$

since

$$V_1 = V_2$$

and

$$w_{\substack{\text{shaft} \\ \text{net out}}} = - w_{\substack{\text{shaft} \\ \text{net in}}}$$

For power loss through the turbine we need to multiply Eq. 1 by the mass flowate, $\dot{m}$, thus

$$\text{power loss} = \dot{m}\left(\frac{P_1 - P_2}{\rho}\right) + \dot{m} g (z_1 - z_2) - \dot{W}_{\substack{\text{shaft} \\ \text{net out}}} \qquad (2)$$

However,

$$\dot{m} = \rho Q = \left(999 \frac{kg}{m^3}\right)\left(4.25 \frac{m^3}{s}\right) = 4246 \frac{kg}{s}$$

Also

$$P_2 = -(0.25\ m\ Hg)(\rho_{Hg})(g) = (0.25\,m)(13.6)\left(999 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)$$

or

$$P_2 = -33{,}300 \frac{N}{m^2}$$

With Eq. 2

$$\text{power loss} = \left(4246 \frac{kg}{s}\right)\frac{\left(415{,}000 \frac{N}{m^2} + 33{,}300 \frac{N}{m^2}\right)}{\left(999 \frac{kg}{m^3}\right)} + \left(4246 \frac{kg}{s}\right)\left(9.81 \frac{m}{s^2}\right)(3\,m)\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right) - \left(1.1 \times 10^6 \frac{N \cdot m}{s}\right)$$

or

$$\text{power loss} = 930{,}000 \frac{N \cdot m}{s} = \underline{\underline{930\ kW}}$$