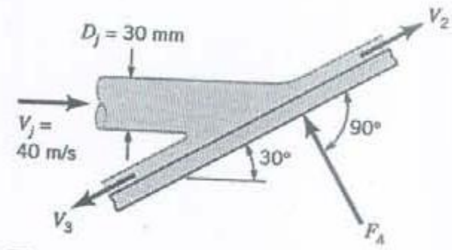
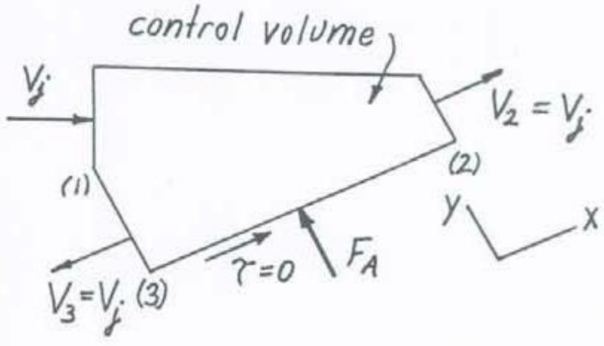
5.57

5.57 A horizontal circular jet of air strikes a stationary flat plate as indicated in Fig. 5.57. The jet velocity is 40 m/s and the jet diameter is 30 mm. If the air velocity magnitude remains constant as the air flows over the plate surface in the directions shown, determine: **(a)** the magnitude of F_A, the anchoring force required to hold the plate stationary; **(b)** the fraction of mass flow along the plate surface in each of the two directions shown; **(c)** the magnitude of F_A, the anchoring force required to allow the plate to move to the right at a constant speed of 10 m/s.



■ FIGURE P5.57



The non-deforming control volume shown in the sketch above is used.

(a) To determine the magnitude of F_A we apply the component of the linear momentum equation (Eq. 5.22) along the direction of F_A. Thus, $\int_{cs} v \rho \vec{V} \cdot \hat{n}\, dA = \Sigma F_y$, or

$$F_A = \dot{m} V_j \sin 30° = \rho A_j V_j V_j \sin 30° = \frac{\rho \pi D_j^2}{4} V_j^2 \sin 30°$$

or

$$F_A = \left(1.23 \frac{kg}{m^3}\right) \frac{\pi (0.030 m)^2}{(4)} \left(40 \frac{m}{s}\right)^2 (\sin 30°) \left(\frac{1 N}{kg \cdot \frac{m}{s^2}}\right) = \underline{\underline{0.696 N}}$$

(b) To determine the fraction of mass flow along the plate surface in each of the 2 directions shown in the sketch above, we apply the component of the linear momentum equation parallel to the surface of the plate, $\int_{cs} u \rho \vec{V} \cdot \hat{n}\, dA = \Sigma F_x$, to obtain

$$R_{\text{along plate surface}} = \dot{m}_2 V_2 - \dot{m}_3 V_3 - \dot{m}_j V_j \cos 30° \qquad (1)$$

(con't)

5.57 (con't)

Since the air velocity magnitude remains constant, the value of $R_{\text{along plate surface}}$ is zero.* Thus from Eq. 1 we obtain

$$\dot{m}_3 V_3 = \dot{m}_2 V_2 - \dot{m}_j V_j \cos 30^\circ \qquad (2)$$

Since $V_3 = V_2 = V_j$, Eq. 2 becomes

$$\dot{m}_3 = \dot{m}_2 - \dot{m}_j \cos 30^\circ \qquad (3)$$

From conservation of mass we conclude that

$$\dot{m}_j = \dot{m}_2 + \dot{m}_3 \qquad (4)$$

Combining Eqs. 3 and 4 we get

$$\dot{m}_3 = \dot{m}_j - \dot{m}_3 - \dot{m}_j \cos 30^\circ$$

or

$$\dot{m}_3 = \dot{m}_j \frac{(1 - \cos 30^\circ)}{2} = \dot{m}_j (0.0670)$$

and

$$\dot{m}_2 = \dot{m}_j (1 - 0.067) = \dot{m}_j (0.933)$$

Thus, $\underline{\underline{\dot{m}_2 \text{ involves } 93.3\% \text{ of } \dot{m}_j}}$ and $\underline{\underline{\dot{m}_3 \text{ involves } 6.7\% \text{ of } \dot{m}_j}}$.

(c) To determine the magnitude of F_A required to allow the plate to move to the right at a constant speed of $10 \frac{m}{s}$, we use a non-deforming control volume like the one in the sketch above that moves to the right with a speed of $10 \frac{m}{s}$. The translating control volume linear momentum equation (Eq. 5.29) leads to

$$F_A = \frac{\rho \pi D_j^2}{4} \left(V_j - 10 \frac{m}{s}\right)^2 \sin 30^\circ$$

or

$$F_A = \left(1.23 \frac{kg}{m^3}\right) \frac{\pi (0.030 m)^2}{4} \left(40 \frac{m}{s} - 10 \frac{m}{s}\right)^2 (\sin 30^\circ) \left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)$$

and

$$F_A = \underline{\underline{0.391\ N}}$$

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* Since $V_1 = V_2 = V_3$ and $p_1 = p_2 = p_3$ and $z_1 = z_2 = z_3$ it follows that the Bernoulli equation is valid from $1 \rightarrow 2$ and $1 \rightarrow 3$.
Thus, there are no viscous effects (Bernoulli equation is valid only for inviscid flow) so that $\tau = 0$. Hence, $R_{\text{along plate}} = 0$.