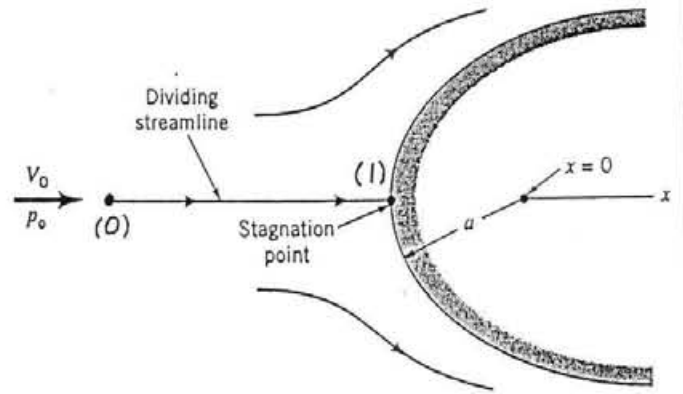


3.5

3.5 An incompressible fluid with density ρ flows steadily past the object shown in Video V3.7 and Fig. P3.5. The fluid velocity along the horizontal dividing streamline ($-\infty \leq x \leq -a$) is found to be $V = V_0(1 + a/x)$, where a is the radius of curvature of the front of the object and V_0 is the upstream velocity. (a) Determine the pressure gradient along this streamline. (b) If the upstream pressure is p_0 , integrate the pressure gradient to obtain the pressure $p(x)$ for $-\infty \leq x \leq -a$. (c) Show from the result of part (b) that the pressure at the stagnation point ($x = -a$) is $p_0 + \rho V_0^2/2$, as expected from the Bernoulli equation.



■ FIGURE P3.5

$$(a) \frac{dp}{ds} = -\rho V \frac{dV}{ds} \quad \text{where } V = V_0 \left(1 + \frac{a}{x}\right)$$

$$\text{Thus, } \frac{dV}{ds} = \frac{dV}{dx} = -\frac{V_0 a}{x^2}$$

or

$$\frac{dp}{ds} = \frac{dp}{dx} = -\rho V_0 \left(1 + \frac{a}{x}\right) \left(-\frac{V_0 a}{x^2}\right) = \underline{\underline{\rho a V_0^2 \left(\frac{1}{x^2} + \frac{a}{x^3}\right)}}$$

$$(b) \int_{p_0}^p dp = \int_{x=-\infty}^x \frac{dp}{dx} dx = \rho a V_0^2 \int_{-\infty}^x \left(\frac{1}{x^2} + \frac{a}{x^3}\right) dx \quad \text{Note: } p = p_0 \text{ at } x = -\infty$$

or

$$p - p_0 = \rho a V_0^2 \left[-\frac{1}{x} - \frac{1}{2} \frac{a}{x^2} \right]_{-\infty}^x$$

Thus,

$$\underline{\underline{p = p_0 - \rho a V_0^2 \left[\frac{1}{x} + \frac{a}{2x^2} \right]}}$$

(c) From part (b), when $x = -a$

$$p \Big|_{x=-a} = p_0 - \rho a V_0^2 \left[-\frac{1}{a} + \frac{a}{2a^2} \right] = \underline{\underline{p_0 + \frac{1}{2} \rho V_0^2}}$$

From the Bernoulli equation $p_0 + \frac{1}{2} \rho V_0^2 = p_1 + \frac{1}{2} \rho V_1^2$

where

$$V_1 = V \Big|_{x=-a} = V_0 \left(1 + \frac{a}{(-a)}\right) = 0$$

Thus, $p_1 = p_0 + \frac{1}{2} \rho V_0^2$ as expected.