2.104

2.104 A 1-m-diameter cylindrical mass, M, is connected to a 2-m-wide rectangular gate as shown in Fig. P2.104. The gate is to open when the water level, h, drops below 2.5 m. Determine the required value for M. Neglect friction at the gate hinge and the pulley.

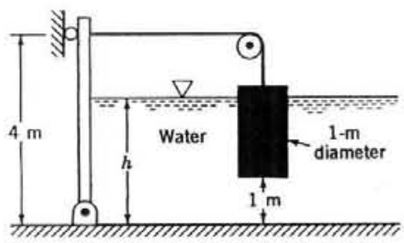
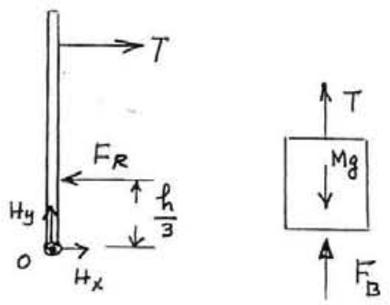


FIGURE P2.104

$$F_R = \gamma h_c A$$
$$= \gamma \left(\frac{h}{2}\right) h (2)$$
$$= \gamma h^2$$

where all lengths are in m.



For equilibrium,

$$\sum M_O = 0$$

so that

$$4T = \left(\frac{h}{3}\right) F_R = \gamma \frac{h^3}{3}$$

and

$$T = \frac{\gamma h^3}{12}$$

For the cylindrical mass $\sum F_{vertical} = 0$ and

$$T = Mg - F_B = Mg - \gamma \forall_{mass}$$

Thus,

$$M = \frac{T + \gamma \forall_{mass}}{g} = \frac{\frac{\gamma h^3}{12} + \gamma \left(\frac{\pi}{4}\right)(1)^2 (h-1)}{g}$$

and for $h = 2.5m$

$$M = \frac{\left(9.80 \times 10^3 \frac{N}{m^3}\right)\left[\frac{(2.5m)^3}{12} + \frac{\pi}{4}(1m)^2(2.5m - 1.0m)\right]}{9.81 \frac{m}{s^2}}$$

$$= \underline{2480\ kg}$$