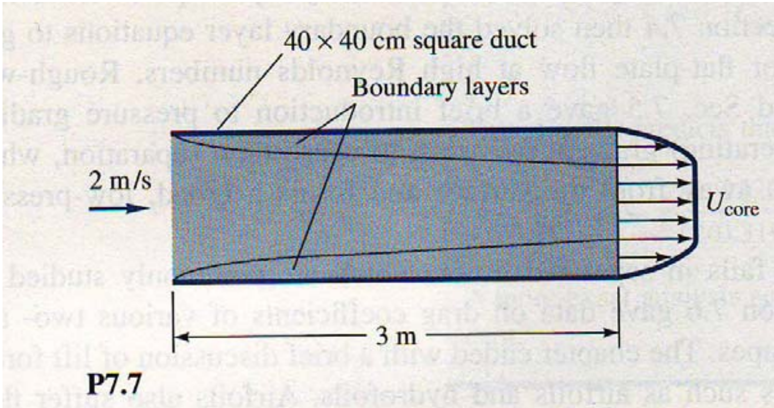
Example: Air at 20°C and 1atm enters a 40-cm-square duct. Using the "displacement thickness" concept, estimate (a) the core velocity and (b) the mean pressure in the core of the flow at the position $x = 3m$. (c) What is the average gradient, in Pa/m, in this section?



Solution:

For air at 20°C, take $\rho = 1.2\, kg/m^3$ and $\mu = 1.8\times10^{-5}\, kg/m\cdot s$. Using laminar boundary layer theory, compute the displacement thickness at $x = 3m$:

$$\mathrm{Re}_x = \frac{\rho U x}{\mu} = \frac{1.2(2)(3)}{1.8\times10^{-5}} = 4\times10^5 \text{ (laminar)}$$

$$\delta^* = \frac{1.721x}{\mathrm{Re}_x^{1/2}} = \frac{1.721(3)}{\left(4\times10^5\right)^{1/2}} \approx 0.0082m$$

Then, by continuity,

$$U_{core} = U_0\left(\frac{L_0}{L_0 - 2\delta^*}\right)^2 = (2.0)\left(\frac{0.4}{0.4-0.0164}\right)^2 \approx 2.175\, m/s$$

The pressure change in the (frictionless) core flow is estimated from Bernoulli's equation. Using gage pressure, then at inlet $p_0 = 0$.

$$p_{core} + \frac{1}{2}\rho U_{core}^2 = p_0 + \frac{1}{2}\rho U_0^2$$

$$p_{core} = 0 + \frac{1}{2}\rho\left(U_0^2 - U_{core}^2\right) = \frac{1}{2}\times1.2\times\left(2^2 - 2.175^2\right) = -0.44Pa$$

The average pressure gradient is

$$\frac{\Delta p}{x} = \frac{-0.44-0}{3.0} \approx -0.15\, Pa/m$$