8.99

8.99 Water is circulated from a large tank, through a filter, and back to the tank as shown in Fig. P8.99. The power added to the water by the pump is 200 ft · lb/s. Determine the flowrate through the filter.

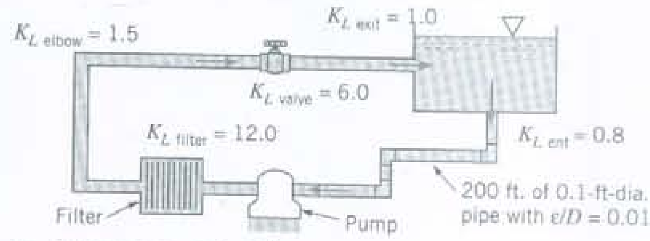

FIGURE P8.99

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + \left(f\frac{\ell}{D} + \sum_i K_{L_i}\right)\frac{V^2}{2g} \qquad (1)$$

where

$p_1 = p_2$, $V_1 = V_2 = 0$, and $z_1 = z_2$

Also, $\dot{W}_p = \gamma Q h_p$ or

$$h_p = \frac{200\ \frac{ft \cdot lb}{s}}{62.4\frac{lb}{ft^3}\left(\frac{\pi}{4}(0.1ft)^2\right)V} = \frac{408}{V}$$

Thus, Eq. (1) becomes

$$\frac{408}{V} = \left(\frac{200\,ft}{0.1\,ft}f + (0.8 + 5(1.5) + 12 + 6 + 1)\right)\frac{V^2}{2(32.2\frac{ft}{s^2})}$$

or

$$V^3 = \frac{13.13}{(f + 0.01365)} \qquad (2)$$

Also,

$$Re = \frac{\rho V D}{\mu} = \frac{1.94\frac{slugs}{ft^3}(V\frac{ft}{s})(0.1ft)}{2.34\times10^{-5}\ \frac{lb \cdot s}{ft^2}} \text{ or } Re = 8290V \qquad (3)$$

Trial and error solution:

Assume $f = 0.04$. From Eq. (2), $V = 6.26\frac{ft}{s}$; from Eq. (3), $Re = 5.20\times10^4$. Thus, from Fig. 8.20, $f = 0.039 \neq 0.04$

Assume $f = 0.039$, or $V = 6.29\frac{ft}{s}$ and $Re = 5.21\times10^4$ and $f = 0.039$ (Checks)

Thus, $Q = AV = \frac{\pi}{4}(0.1ft)^2(6.29\frac{ft}{s}) = \underline{\underline{0.0494\frac{ft^3}{s}}}$

Alternatively, the Colebrook equation (Eq. 8.35) could be used rather than the Moody chart. Thus,

(con't)

8.99 (con't)

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right), \text{ where from Eq.(2),} \qquad (4)$$

$$f = (13.13/V^3) - 0.01365 \qquad (5)$$

Thus, by combining Eqs. (3), (4), and (5) we obtain the following equation for V:

$$1/[(13.13/V^3) - 0.01365]^{\frac{1}{2}} = -2.0 \log\left\{\frac{0.01}{3.7} + 2.51/[(8290V)((13.13/V^3) - 0.01365)^{\frac{1}{2}}]\right\} \qquad (6)$$

Using a computer root-finding program gives the solution to Eq.(6) as $V = 6.29 \frac{ft}{s}$, the same as obtained by the above trial and error method.