8.72

8.72 Given two rectangular ducts with equal cross-sectional area, but different aspect ratios (width/height) of 2 and 4, which will have the greater frictional losses? Explain your answer.

The duct with the greater losses is the one with the largest headloss per length, h_L/ℓ, where $h_L = f\,\dfrac{\ell}{D_h}\,\dfrac{V^2}{2g}$. If the areas are equal, then the velocities are equal since $V = Q/A$.

Let $(\;)_2$ and $(\;)_4$ denote ducts with aspect ratios of 2 and 4, respectively. Thus,

$$(h_L/\ell)_4 = \frac{f_4}{D_{h4}}\frac{V_4^2}{2g} \quad \text{and} \quad (h_L/\ell)_2 = \frac{f_2}{D_{h2}}\frac{V_2^2}{2g}, \quad \text{where } V_2 = V_4.$$

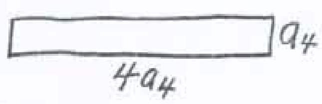
Hence,

$$(h_L/\ell)_4 \Big/ (h_L/\ell)_2 = \frac{f_4}{D_{h4}} \Big/ \frac{f_2}{D_{h2}} = \frac{f_4}{f_2}\,\frac{D_{h2}}{D_{h4}} \qquad (1)$$

Let $A_2 = (2a)\,a$

and

$A_4 = (4a_4)\,a_4$



Thus, since $A_2 = A_4$,

$$2a^2 = 4a_4^2, \quad \text{or} \quad a_4 = \frac{1}{\sqrt{2}}\,a$$

and

$$D_{h_2} = 4A_2/P_2 = 4(2a^2)/[4a+2a] = \frac{4}{3}a = 1.33\,a \qquad (2)$$

and

$$D_{h4} = 4A_4/P_4 = 4(2a^2)/[\tfrac{2}{\sqrt{2}}a + \tfrac{8}{\sqrt{2}}a] = \frac{4\sqrt{2}}{5}a = 1.13\,a \qquad (3)$$

so that

$$\frac{D_{h2}}{D_{h4}} = \frac{\frac{4}{3}a}{\frac{4\sqrt{2}}{5}a} = \frac{5}{3\sqrt{2}} = 1.179 \quad \text{so that Eq. (1) becomes}$$

$$(h_L/\ell)_4 / (h_L/\ell)_2 = 1.179\,\frac{f_4}{f_2} \qquad (4)$$

In general, $f = f(Re, \frac{\varepsilon}{D})$ in such a way that if $\frac{\varepsilon}{D}$ increases, f increases and if Re decreases, f increases. This is seen from the Moody chart as indicated below.

(con't)