8.50

8.50 Two equal length, horizontal pipes, one with a diameter of 1 in., the other with a diameter of 2 in., are made of the same material and carry the same fluid at the same flow rate. Which pipe produces the larger head loss? Justify your answer.

For either pipe $h_L = f\frac{\ell}{D}\frac{V^2}{2g}$, where $V = Q/A = Q/(\frac{\pi}{4}D^2)$.

Thus,

$$h_L = f\frac{\ell}{D}\left[4Q/(\pi D^2)\right]^2/2g = \frac{8}{\pi^2}f\frac{\ell}{D^5}Q^2/g$$

or

$$h_L = \left[\frac{8}{\pi^2}\frac{\ell Q^2}{g}\right]\frac{f}{D^5} \qquad (1)$$

Let $(\,)_1$ and $(\,)_2$ denote the 1 in. and 2 in. diameter pipes, respectively. Thus, with $Q_1 = Q_2$ and $\ell_1 = \ell_2$, Eq. (1) gives

$$\frac{h_{L1}}{h_{L2}} = \frac{(f_1/D_1^5)}{(f_2/D_2^5)} = \left(\frac{f_1}{f_2}\right)\left(\frac{D_2}{D_1}\right)^5 = \left(\frac{f_1}{f_2}\right)\left(\frac{2\,in.}{1\,in.}\right)^5$$

or

$$\frac{h_{L1}}{h_{L2}} = 32\left(\frac{f_1}{f_2}\right) \qquad (2)$$

Although $f_1 \neq f_2$ (because $Re_1 \neq Re_2$ and $\varepsilon/D_1 \neq \varepsilon/D_2$) the ratio f_1/f_2 would not be significantly different than 1, especially compared to the factor of 32 in Eq. (2). For example, assume $Re_1 = 10{,}000$ and $\varepsilon/D_1 = 0.001$ so that $f_1 = 0.033$ (see Fig. 8.20). Thus, since

$Re = VD/\nu = (Q/\frac{\pi}{4}D^2)D/\nu = \frac{4Q}{\pi\nu}/D$ it follows that if $Re_1 = 10{,}000$, then $Re_2 = 5{,}000$ and $\varepsilon/D_2 = 0.0005$ if $\varepsilon/D_1 = 0.001$. Hence, $f_2 = 0.037$ so that $h_{L1}/h_{L2} = 32\,(0.033/0.037) = 28.5 >> 1$. Similar results would be true for other Re, ε/D values.

Thus, $h_{L1}/h_{L2} = 32\,(f_1/f_2) > 1$, The <u>smaller pipe</u> has the larger head loss.