7.16

7. The pressure rise, Δp, across a pump can be expressed as

$$\Delta p = f(D, \rho, \omega, Q)$$

where D is the impeller diameter, ρ the fluid density, ω the rotational speed, and Q the flowrate. Determine a suitable set of dimensionless parameters.

$\Delta p \doteq FL^{-2}$ $\quad D \doteq L$ $\quad \rho \doteq FL^{-4}T^{2}$ $\quad \omega \doteq T^{-1}$ $\quad Q = L^{3}T^{-1}$

From the pi theorem, $5-3=2$ pi terms required. Use D, ρ, and ω as repeating variables. Thus,

$$\Pi_1 = \Delta p\, D^a \rho^b \omega^c$$

and so that

$$(FL^{-2})(L)^a(FL^{-4}T^2)^b(T^{-1})^c \doteq F^0L^0T^0$$

$$1 + b = 0 \qquad \text{(for } F\text{)}$$

$$-2 + a - 4b = 0 \qquad \text{(for } L\text{)}$$

$$2b - c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=-2$, $b=-1$, $c=-2$, and therefore

$$\Pi_1 = \frac{\Delta p}{D^2 \rho \omega^2}$$

Check dimensions using MLT system:

$$\frac{\Delta p}{D^2\rho\omega^2} \doteq \frac{ML^{-1}T^{-2}}{(L)^2(ML^{-3})(T^{-1})^2} \doteq M^0L^0T^0 \qquad \therefore \text{ok}$$

For Π_2:

$$\Pi_2 = Q\, D^a \rho^b \omega^c$$

$$(L^3T^{-1})(L)^a(FL^{-4}T^2)^b(T^{-1})^c = F^0L^0T^0$$

$$b = 0 \qquad \text{(for } F\text{)}$$

$$3 + a - 4b = 0 \qquad \text{(for } L\text{)}$$

$$-1 + 2b - c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=-3$, $b=0$, $c=-1$, and therefore

$$\Pi_2 = \frac{Q}{D^3\omega}$$

Check dimensions using MLT system:

$$\frac{Q}{D^3\omega} \doteq \frac{L^3T^{-1}}{(L)^3(T^{-1})} \doteq M^0L^0T^0 \qquad \therefore \text{ok}$$

Thus,

$$\underline{\underline{\frac{\Delta p}{D^2\rho\omega^2} = \phi\left(\frac{Q}{D^3\omega}\right)}}$$