6.15

6.15 For each of the following stream functions, with units of m^2/s, determine the magnitude and the angle the velocity vector makes with the x-axis at $x = 1$ m, $y = 2$ m. Locate any stagnation points in the flow field.
(a) $\psi = xy$
(b) $\psi = -2x^2 + y$

From the definition of the stream function,

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \qquad \text{(Eqs. 6.37)}$$

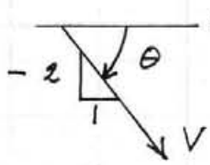
(a) For $\psi = xy$,

$$u = \frac{\partial \psi}{\partial y} = x \qquad v = -\frac{\partial \psi}{\partial x} = -y$$

At $x = 1m$, $y = 2m$, it follows that $u = 1\frac{m}{s}$ and $v = -2\frac{m}{s}$

Thus,

$$|V| = \sqrt{u^2 + v^2} = \sqrt{(1m)^2 + (-2m)^2} = \underline{2.24\frac{m}{s}}$$



$$\tan\theta = \frac{-2}{1} \qquad \theta = \underline{-63.4°}$$

Since $u = 0$ at $x = 0$ and $v = 0$ at $y = 0$, a stagnation point occurs at $x = y = 0$.

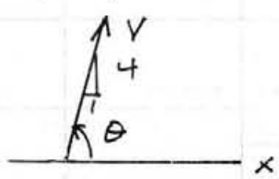
(b) For $\psi = -2x^2 + y$,

$$u = \frac{\partial \psi}{\partial y} = 1\frac{m}{s} \qquad v = -\frac{\partial \psi}{\partial x} = 4x$$

At $x = 1m$, $y = 2m$, it follows that $u = 1\frac{m}{s}$ and $v = 4\frac{m}{s}$

Thus,

$$|V| = \sqrt{u^2 + v^2} = \sqrt{(1\frac{m}{s})^2 + (4\frac{m}{s})^2} = \underline{4.12\frac{m}{s}}$$



$$\tan\theta = \frac{4}{1} \qquad \underline{\theta = 76.0°}$$

Since $u \neq 0$, there are no stagnation points.