6.5

6.5 Determine the vorticity field for the following velocity vector:

$$\mathbf{V} = (x^2 - y^2)\hat{\mathbf{i}} - 2xy\hat{\mathbf{j}}$$

$$\nabla \times \vec{V} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\hat{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k},$$

where

$u = x^2 - y^2$, $v = -2xy$, and $w = 0$

Thus,

$$\nabla \times \vec{V} = 0\hat{i} + 0\hat{j} + \left[\frac{\partial}{\partial x}(-2xy) - \frac{\partial}{\partial y}(x^2 - y^2)\right]\hat{k}$$

$$= [-2y - (-2y)]\hat{k} = 0\hat{k}$$

Hence,

$$\nabla \times \vec{V} = \underline{\underline{0}}$$