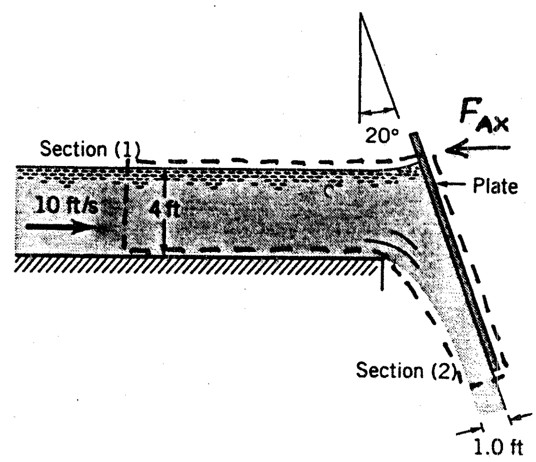
5.48

5.48 Water flows from a two-dimensional open channel and is diverted by an inclined plate as illustrated in Fig. P5.48. When the velocity at section (1) is 10 ft/s, what horizontal force (per unit width) is required to hold the plate in position? At section (1) the pressure distribution is hydrostatic, and the fluid acts as a free jet at section (2). Neglect friction.



■ FIGURE P5.48

A control volume that contains most of the plate and the water being turned by the plate as shown in the sketch above is used. Application of the horizontal x-direction component of the linear momentum equation yields

$$-V_1 \rho V_1 A_1 + V_2 \sin 20^\circ \rho V_2 A_2 = -F_{Ax} + \frac{1}{2}\gamma_w h_1 A_1 \qquad (1)$$

From conservation of mass we obtain

$$V_2 = \frac{A_1}{A_2} V_1 = \frac{h_1}{h_2} V_1$$

Thus, Eq. 1 becomes for unit width

$$-V_1^2 \rho h_1 + \left(\frac{h_1}{h_2} V_1\right)^2 \sin 20^\circ \rho h_2 = -F_{Ax} + \frac{1}{2}\gamma_w h_1^2$$

or

$$F_{Ax} = \frac{1}{2}\gamma_w h_1^2 + V_1^2 \rho h_1 - \left(\frac{h_1}{h_2} V_1\right)^2 \sin 20^\circ \rho h_2$$

Then

$$F_{Ax} = \frac{1}{2}\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)(4\text{ ft})^2 + \left(10\,\frac{\text{ft}}{\text{s}}\right)^2\left(1.94\,\frac{\text{slugs}}{\text{ft}^3}\right)\left(1\,\frac{\text{lb}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}\right)(4\text{ ft})$$
$$-\left[\left(\frac{4\text{ ft}}{1\text{ ft}}\right)\left(10\,\frac{\text{ft}}{\text{s}}\right)\right]^2 \sin 20^\circ \left(1.94\,\frac{\text{slugs}}{\text{ft}^3}\right)\left(1\,\frac{\text{lb}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}\right)(1\text{ ft})$$

and

$$F_{Ax} = \underline{\underline{213}}\text{ lb}$$