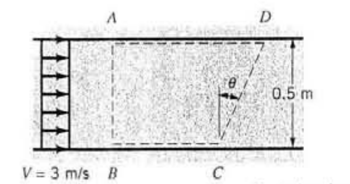
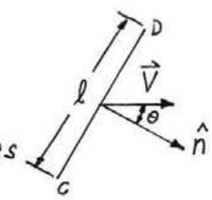
4.71

4.71 Water flows through the 2-m-wide rectangular channel shown in Fig. P4.71 with a uniform velocity of 3 m/s. (**a**) Directly integrate Eq. 4.16 with $b = 1$ to determine the mass flowrate (kg/s) across section CD of the control volume. (**b**) Repeat part (a) with $b = 1/\rho$, where ρ is the density. Explain the physical interpretation of the answer to part (**b**).



■ FIGURE P4.71

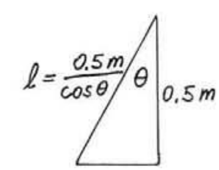
a) $$\dot{B}_{out} = \int_{cs_{out}} \rho b \, \vec{V}\cdot\hat{n} \, dA \qquad (1)$$



With $b = 1$ and $\vec{V}\cdot\hat{n} = V\cos\theta$ this becomes

$$\dot{B}_{out} = \int_{CD} \rho V \cos\theta \, dA = \rho V \cos\theta \int_{CD} dA$$

$$= \rho V \cos\theta \, A_{CD}, \quad \text{where } A_{CD} = \ell \, (2m)$$



$$= \left(\frac{0.5\,m}{\cos\theta}\right)(2m)$$

$$= \left(\frac{1}{\cos\theta}\right) m^2$$

Thus, with $V = 3\,m/s$,

$$\dot{B}_{out} = \left(3\,\tfrac{m}{s}\right)\cos\theta\left(\frac{1}{\cos\theta}\right)m^2\left(999\,\tfrac{kg}{m^3}\right) = \underline{\underline{3000\,\tfrac{kg}{s}}}$$

b) With $b = 1/\rho$ Eq. (1) becomes

$$\dot{B}_{out} = \int_{CD} \vec{V}\cdot\hat{n} \, dA = \int_{CD} V\cos\theta \, dA = V\cos\theta \, A_{CD}$$

$$= \left(3\,\tfrac{m}{s}\right)\cos\theta\left(\frac{1}{\cos\theta}\right)m^2 = \underline{\underline{3.00\,\tfrac{m^3}{s}}}$$

With $b = 1/\rho = \dfrac{1}{\left(\frac{mass}{vol}\right)} = \dfrac{vol}{mass}$ it follows that "B = volume" (i.e., $b = \frac{B}{mass}$) so that $\int \vec{V}\cdot\hat{n}\,dA = \dot{B}_{out}$ represents the volume flowrate (m^3/s) from the control volume.