4.66 A layer of oil flows down a vertical plate as shown in Fig. P4..66 with a velocity of $\mathbf{V} = (V_0/h^2)(2hx - x^2)\hat{\mathbf{j}}$ where V_0 and h are constants. **(a)** Show that the fluid sticks to the plate and that the shear stress at the edge of the layer $(x = h)$ is zero. **(b)** Determine the flowrate across surface AB. Assume the width of the plate is b. (Note: The velocity profile for laminar flow in a pipe has a similar shape. See Video V6.13)
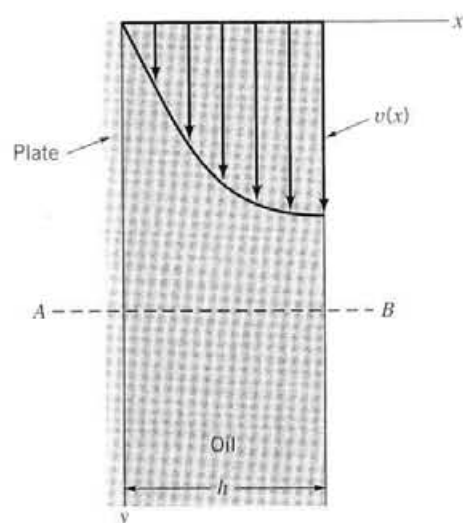

■ FIGURE P4.66

a) $\quad v = \dfrac{V_0}{h^2}(2hx - x^2)$

Thus,

$$v\bigg|_{x=0} = \frac{V_0}{h^2}(0-0) = 0 \quad \text{and}$$

$$\tau\bigg|_{x=h} = \mu\frac{dv}{dx}\bigg|_{x=h} = \mu\frac{V_0}{h^2}\Big[2h - 2x\Big|_{x=h} = 0$$

Hence, the fluid sticks to the plate and there is no shear stress at the free surface.

b) $\quad Q_{AB} = \displaystyle\int v\, dA = \int_{x=0}^{x=h} v\, b\, dx = \int_{0}^{h} \frac{V_0}{h^2}(2hx - x^2)\, b\, dx$

or

$$Q_{AB} = \frac{V_0 b}{h^2}\Big[hx^2 - \tfrac{1}{3}x^3\Big|_{0}^{h} = \underline{\underline{\tfrac{2}{3}V_0 h b}}$$