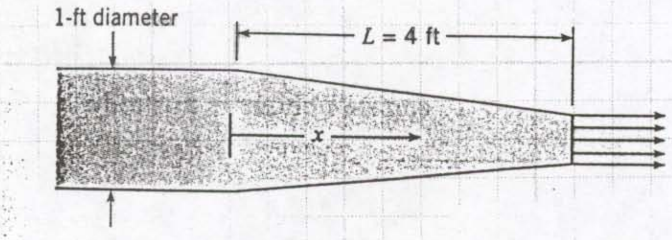
4.51 The velocity of water flow in the nozzle shown is given by the following expression: $V = 2t/(1 - 0.5x/L)^2$, where V = velocity in feet per second, t = time in seconds, x = distance along the nozzle, and L = length of nozzle = 4 ft. When $x = 0.5L$ and $t = 3$ s, what is the local acceleration along the centerline? What is the convective acceleration? Assume one-dimensional flow prevails.



$\underline{V} = u\hat{i} + v\hat{j} + w\hat{k}$ $\qquad \frac{D\underline{V}}{Dt} = \frac{\partial \underline{V}}{\partial t} + \underline{V}\cdot\nabla\underline{V}$

$\underline{V} = u(x,t)\hat{i}$ $\qquad \boxed{a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}}$

$u(x,t) = \frac{2t}{(1-\frac{.5x}{L})^2}$ $\qquad a_y = a_z = 0$

$\frac{\partial u}{\partial t} = \frac{2}{(1-\frac{.5x}{L})^2}$ $\qquad \frac{\partial u}{\partial x} = 2t\cdot(-2)\left(1-\frac{.5x}{L}\right)^{-3}\cdot -\frac{.5}{L}$

$a_{local} = \frac{\partial u}{\partial t}\Big|_{\substack{x=.5L \\ t=3s}} = \frac{2}{(1-.5^2)^2} = 3.56\ \frac{ft}{s^2}$

$a_{conv} = u\frac{\partial u}{\partial x} = \frac{2t}{(1-\frac{.5x}{L})^2}\cdot\frac{2t}{(1-\frac{.5x}{L})^3 L} = \frac{4t^2}{L(1-\frac{.5x}{L})^5}\Big|_{\substack{x=.5L \\ t=3s}}$

$= 37.92\ \frac{ft}{s^2}$