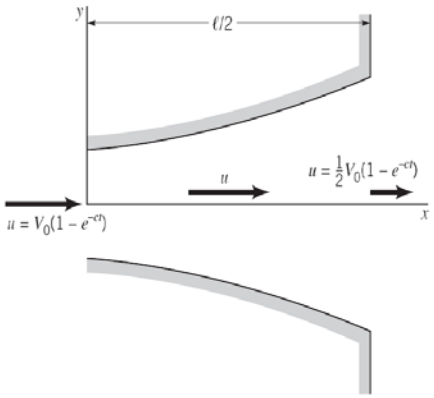
October 6, 2014

NAME

Fluids-ID

Quiz 5. As a valve is opened, water flows through the diffuser at an increasing flow rate so that the velocity along the centerline is given by



$$\underline{V} = u\hat{\imath} = V_0(1-e^{-ct})\left(1-\frac{x}{\ell}\right)\hat{\imath}$$

where V_0, c, and ℓ are constants. If V_0 = 10 ft/s, c = 0.5 1/s and ℓ = 5 ft, determine the acceleration at $x = \ell/2$ = 2.5 ft and t = 2 s.

- Acceleration: $\underline{a} = a_x\hat{\imath} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)\hat{\imath}$

Note: Attendance (+2 points), format (+1 point)

Solution:

With $u = V_0(1-e^{-ct})\left(1-\frac{x}{\ell}\right)$, $v = 0$, and $w = 0$,

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$$ (+4 points)

where,

$$\frac{\partial u}{\partial t} = V_0\left(1-\frac{x}{\ell}\right)c\cdot e^{-ct}$$
$$\frac{\partial u}{\partial x} = V_0(1-e^{-ct})\left(-\frac{1}{\ell}\right)$$

Thus,

$$a_x = V_0\left(1-\frac{x}{\ell}\right)\left[ce^{-ct} - \frac{V_0}{\ell}(1-e^{-ct})^2\right]$$ (+2 points)

At x = 2.5 ft and t = 2 s,

$$a_x = \left(10\frac{\text{ft}}{\text{s}}\right)\left(1-\frac{2.5\text{ ft}}{5\text{ ft}}\right)\left[0.5e^{-\frac{0.5}{\text{s}}\times 2\text{ s}} - \frac{10\text{ ft/s}}{5\text{ ft}}\left(1-e^{-\frac{0.5}{\text{s}}\times 2\text{ s}}\right)^2\right] = -3.08\,\text{ft/s}^2$$

(+1 point)