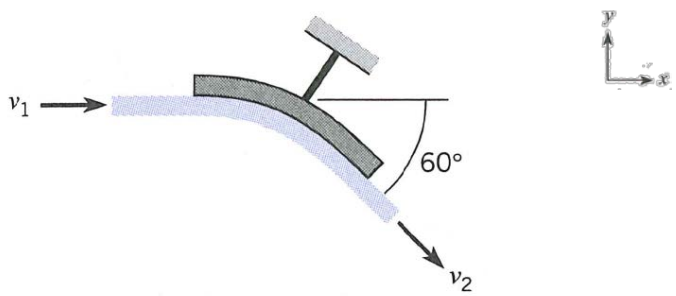
57:020 Mechanics of Fluids and Transport

October 23, 2013

NAME

Fluids-ID

Quiz 8. A water jet (ρ = 1.94 slug/ft^3) is deflected 60° by a stationary vane as shown in the figure. The incoming jet has a speed of 100 ft/s and a diameter of 1 in. Neglect the influence of gravity and assume steady and irrotational flow.



Momentum equation:

$$\sum \underline{\boldsymbol{F}} = \int_{CV} \frac{\partial}{\partial t}(\underline{\boldsymbol{V}}\rho)d\mathcal{V} + \int_{CS} \underline{\boldsymbol{V}}\rho\underline{\boldsymbol{V}} \cdot d\underline{\boldsymbol{A}}$$

Bernoulli equation:

$$p_1 + \frac{\rho v_1^2}{2} + \gamma z_1 = p_2 + \frac{\rho v_2^2}{2} + \gamma z_2$$

(1) Show that $v_1 = v_2$ using the Bernoulli equation.

(2) Find the relationship between cross-sectional area at location 1 (A_1) and 2 (A_2) using the continuity equation.

(3) Find the horizontal F_x and vertical F_y components of the force exerted by the jet on the vane.

Note: Attendance (+2 points), format (+1 point)

<u>Solution:</u>

Bernoulli equation: $\cancel{p_1} + \frac{1}{2}\rho v_1^2 + \gamma \cancel{z_1} = \cancel{p_2} + \frac{1}{2}\rho v_2^2 + \gamma \cancel{z_2}$ $\quad \therefore v_1 = v_2 = v$ (+2 points)

Continuity equation: $v_1 A_1 = v_2 A_2 \quad \therefore A_1 = A_2 = A$ (+1 point)

x-Momentum equation:

$$F_x = v_1(-\rho v_1 A_1) + v_2 \cos 60^\circ \,(\rho v_2 A_2) = -\rho A v^2 (1 - \cos 60^\circ)$$

$$= -\left(1.94 \frac{\text{slug}}{\text{ft}^3}\right) \frac{\pi}{4} \left(\frac{1}{12}\text{ft}\right)^2 \left(100 \frac{\text{ft}}{\text{s}}\right)^2 (1 - \cos 60^\circ) = -53.0 \text{ lbf}$$ (+2 points)

y-Momentum equation:

$$F_y = -v_2 \sin 60^\circ \,(\rho v_2 A_2) = -\rho A v^2 \sin 60^\circ$$

$$= -\left(1.94 \frac{\text{slug}}{\text{ft}^3}\right) \frac{\pi}{4} \left(\frac{1}{12}\text{ft}\right)^2 \left(100 \frac{\text{ft}}{\text{s}}\right)^2 (\sin 60^\circ) = -91.8 \text{ lbf}$$ (+2 points)

The force of the jet on the vane is opposite in direction to the force required to hold the vane stationary.