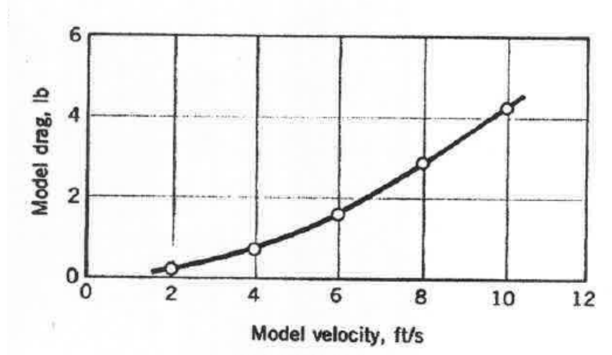
NAME

Fluids-ID

Quiz 11.

A. The drag D on a sphere moving in a fluid is known to be function of the sphere diameter d, the velocity V, and the fluid viscosity μ and density ρ. Using the pi theorem, find an appropriate dimensionless relationship.

B. Laboratory tests on a 4-in-diameter sphere model were performed in a water tunnel and some model data are plotted in the Figure. Estimate the prototype drag on a 8-ft-diameter balloon moving in air at a velocity 3.28 ft/s. (Hint: You will need to set Π parameters for the tests and the balloon equal to each other)



Notes:

- $D \doteq F; d \doteq L; V \doteq LT^{-1}; \rho \doteq FL^{-4}T^2; \mu \doteq FL^{-2}T$
- For water: $\mu_m = 2.3 \times 10^{-5}$ lb·s/ft^2 and $\rho_m = 1.94$ slug/ft^3
- For air: $\mu = 3.7 \times 10^{-7}$ lb·s/ft^2 and $\rho = 2.38 \times 10^{-3}$ slug/ft^3

Attendance (+2 points), format (+1 point)

Solution:

A. From the Buckingham Pi theorem, $k - r = 5 - 3 = 2$ Π term is needed. Use μ and D as repeated variables.

First Π parameter

$$\Pi_1 = \mu d^a V^b \rho^c$$

Units

$$F^0 L^0 T^0 \doteq (FL^{-2}T)(L)^a (LT^{-1})^b (FL^{-4}T^2)^c$$

or

$$F^0 L^0 T^0 \doteq F^{(1+c)} L^{(-2+a+b-4c)} T^{(1-b+2c)}$$

To be dimensionless it follows that

$$\begin{aligned} F: &\quad 1 + c &= 0 \\ L: &\quad -2 + a + b - 4c &= 0 \\ T: &\quad 1 - b + 2c &= 0 \end{aligned}$$

Therefore, a = -1, b = -1, c = -1.

$$\Pi_1 = \frac{\mu}{\rho V d} \left(\text{or } \frac{\rho V d}{\mu}\right) \qquad \text{(+2 points)}$$

Second Π parameter

$$\Pi_2 = D d^a V^b \rho^c$$

Units

$$F^0L^0T^0 \doteq (F)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c$$

or

$$F^0L^0T^0 \doteq F^{(1+c)}L^{(a+b-4c)}T^{(-b+2c)}$$

To be dimensionless it follows that

$$\begin{aligned} F&: \quad 1+c &= 0 \\ L&: \quad a+b-4c &= 0 \\ T&: \quad -b+2c &= 0 \end{aligned}$$

Therefore, a = -2, b = -2, c = -1.

$$\Pi_2 = \frac{D}{\rho V^2 d^2} \qquad \text{(+2points)}$$

Thus, the functional relationship must be of the from

$$\frac{D}{\rho V^2 d^2} = \phi\left(\frac{\rho V d}{\mu}\right)$$

B. From Re similarity

$$\frac{\rho_m V_m d_m}{\mu_m} = \frac{\rho V d}{\mu}$$

or

$$V_m = \left(\frac{\mu_m}{\mu}\right)\left(\frac{\rho}{\rho_m}\right)\left(\frac{d}{d_m}\right)V$$

$$V_m = \left(\frac{2.3\times10^{-5}}{3.7\times10^{-7}}\right)\left(\frac{2.38\times10^{-3}}{1.94}\right)\left(\frac{8}{4/12}\right)(3.28) = 6\ ft/s \qquad \text{(+1 point)}$$

From the graph for $V_m = 6\ ft/s$ $D_m = 1.75\ lb$ (+1 point)

$$\frac{D}{\rho V^2 d^2} = \frac{D_m}{\rho_m V_m^2 d_m^2}$$

or

$$D = \left(\frac{\rho}{\rho_m}\right)\left(\frac{V}{V_m}\right)^2\left(\frac{d}{d_m}\right)^2 D_m$$

Therefore

$$D = \left(\frac{2.38\times10^{-3}}{1.94}\right)\left(\frac{3.28}{6}\right)^2\left(\frac{8}{4/12}\right)^2(2) = 0.37\ lb \quad \text{(+1 point)}$$