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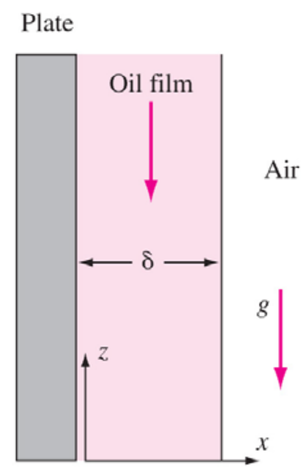
NAME

Fluids-ID



Quiz 10. An oil film drains steadily down the side of a vertical wall, as shown on the Figure. After an initial development at the top of the wall, the film becomes independent of z and of constant thickness (δ). Assume that $w = w(x)$, pressure gradient is negligible, and shear stress (τ) at the free surface is zero.

A. Solve Navier-Stokes for $w(x)$.
B. If the oil is SAE 30W (ρ = 891 kg/m^3 and μ = 0.29 kg/m·s), δ = 2 mm, and the plate width (into the paper) W=1 m and height H=2 m, find (a) the maximum velocity w_{max}, (b) flow rate Q, (c) average velocity $\overline{w}$, (d) shear stress on the wall τ_w, and (e) the friction drag force acting on the plate D.

Continuity:	$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$
Momentum:	$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{dp}{dz}-\rho g+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$
Flow rate:	$Q=\int_A \underline{V}\cdot\underline{dA}$
Average velocity:	$\overline{w}=Q/A$
Shear stress:	$\tau=\mu\frac{dw}{dx}$
Friction drag:	$D=\tau_w\cdot S$, where S = wetted area

Note: Attendance (+2 points), format (+1 point)

Part A:

The assumption of parallel flow, $u = v = 0$ and $w = w(x)$, satisfies continuity and makes the x and z momentum equations irrelevant. We are left with the z momentum equation

$$\rho\left(0+0\times\frac{\partial w}{\partial x}+0\times\frac{\partial w}{\partial y}+w\times 0\right)=-(0)-\rho g+\mu\left(\frac{\partial^2 w}{\partial x^2}+0+0\right)$$

There no convective acceleration and the pressure gradient is negligible due to the free surface. We are left with a second order linear differential equation for $w(x)$

$$\frac{d^2 w}{dx^2}=\frac{\rho g}{\mu}$$

Integrating

$$\frac{dw}{dx} = \frac{\rho g}{\mu} x + C_1$$

$$w = \frac{\rho g}{2\mu} x^2 + C_1 x + C_2$$

At the free surface, $\tau(\delta) = \mu \frac{dw}{dx} = 0$, or $\left.\frac{dw}{dx}\right)_{x=\delta} = 0$, hence $C_1 = -\rho g \delta / \mu$

At the wall, $w(0) = 0 = C_2$

Therefore

$w = \frac{\rho g}{2\mu} x^2 - \frac{\rho g \delta}{\mu} x = \frac{\rho g}{2\mu}(x^2 - 2\delta x)$ (+4.5 points)

<u>Part B:</u>

(a) Maximum velocity is where $\frac{dw}{dx} = \frac{\rho g}{2\mu}(2x - 2\delta) = 0$ or $x = \delta$, thus

$w_{max} = w(\delta) = -\frac{\rho g \delta^2}{2\mu} = -\frac{(891)(9.81)(0.002)^2}{(2)(0.29)} = -0.06\ m/s$ (+0.5 point)

(b) Flow rate is

$$Q = \int_0^\delta w(x) \cdot (-W dx) = -W \cdot \left[\frac{\rho g}{2\mu}\left(\frac{x^3}{3} - \delta x^2\right)\right]_0^\delta = \frac{\rho g \delta^3 W}{3\mu}$$

$\therefore Q = \frac{(891)(9.81)(0.002)^3(1)}{(3)(0.29)} = 8.04 \times 10^{-5} m^3/s$ (+0.5 point)

(c) Average velocity is

$\overline{w} = \frac{Q}{A} = \frac{Q}{W \cdot \delta} = \frac{8.04 \times 10^{-5}}{(1)(0.002)} = 0.04\ m/s$ (+0.5 point)

(d) Wall shear stress is

$$\tau_w = \mu \left.\frac{dw}{dx}\right)_{x=0} = \mu \cdot \left.\frac{\rho g}{2\mu}(2x - 2\delta)\right)_{x=0} = -\rho g \delta = -(891)(9.81)(0.002) = -17.48\ N/m^2$$

(+0.5 point)

(e) Friction drag is

$D = \tau_w \cdot S = \tau \cdot (W \cdot H) = (-17.48)(1)(2) = -35\ N$ (+0.5 point)