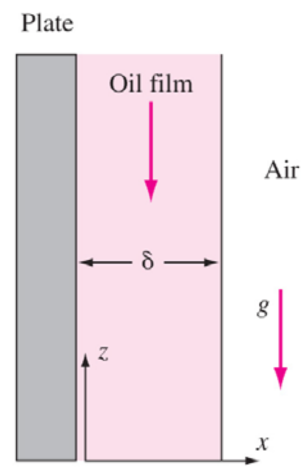
November 4, 2013

NAME	
Fluids-ID	

Quiz 10. An oil film drains steadily down the side of a vertical wall, as shown on the Figure. After an initial development at the top of the wall, the film becomes independent of z and of constant thickness (δ). Assume that $w = w(x)$, pressure gradient is negligible, and shear stress (τ) at the free surface is zero.



A. Solve Navier-Stokes for $w(x)$.

B. If the oil is SAE 30W (ρ = 891 kg/m^3 and μ = 0.29 kg/m·s), δ = 2 mm, and the plate width (into the paper) W=1 m and height H=2 m, find (a) the maximum velocity w_{max}, (b) flow rate Q, (c) average velocity $\overline{w}$, (d) shear stress on the wall τ_w, and (e) the friction drag force acting on the plate D.

Continuity:	$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$
Momentum:	$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{dp}{dz}-\rho g+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$
Flow rate:	$Q=\int_A \underline{V}\cdot \underline{dA}$
Average velocity:	$\overline{w}=Q/A$
Shear stress:	$\tau=\mu\frac{dw}{dx}$
Friction drag:	$D=\tau_w\cdot S$, where $S=$ wetted area

Note: Attendance (+2 points), format (+1 point)