57:020 Mechanics of Fluids and Transport

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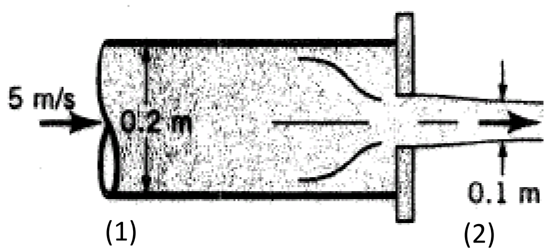
NAME

Fluids-ID

Quiz 8. The exit plane of a 0.20 m diameter pipe is partially blocked by a plate with a hole in it that produces a 0.10 m diameter stream as shown in the figure. The water velocity in the pipe is 5 m/s. Gravity and viscous effects are negligible. Determine (a) the pressure at inlet by using Bernoulli's equation and the conservation of mass, (b) the force needed to hold the plate against the pipe.

Hint:

1) Gravity is negligible.
2) Flow is incompressible, steady flow.
3) Density of water, $\rho = 998\ kg/m^3$
4) Pressure at (2), $p_2 = p_{atm}$



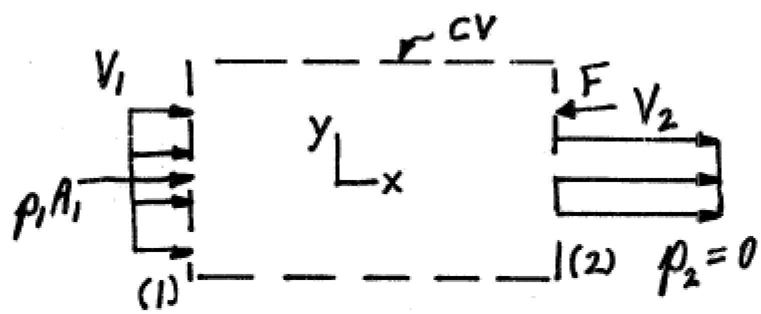
Momentum equation:

$$\sum \underline{\boldsymbol{F}} = \frac{\partial}{\partial t}\int_{CV} \underline{\boldsymbol{V}}\rho d\forall + \int_{CS} \underline{\boldsymbol{V}}\rho \underline{\boldsymbol{V}} \cdot d\underline{\boldsymbol{A}}$$

Bernoulli's equation:

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

Solution:



(a) Assuming $z_1 = z_2$ and $p_2 = 0$, Bernoulli's equation reduces to

$$p_1 + \frac{1}{2}\rho V_1^2 = \frac{1}{2}\rho V_2^2$$

(+2 points)

By use of conservation of mass

$$V_1 A_1 = V_2 A_2$$

Thus,

$$V_2 = \frac{A_1}{A_2} V_1 = \frac{\frac{\pi}{4}(0.2m)^2}{\frac{\pi}{4}(0.1m)^2} \times (5\ \mathrm{m/s}) = 20\ \mathrm{m/s}$$

(+2 points)

Therefore,

$$p_1 = \frac{1}{2}\rho(V_2^2 - V_1^2) = \frac{1}{2}\left(998\frac{kg}{m^3}\right)\left(\left(20\frac{m}{s}\right)^2 - \left(5\frac{m}{s}\right)^2\right) = 187\ kPa$$

(+1 points)

(b) Linear momentum equation

$$\sum F = p_1 A_1 - F = V_2 \rho V_2 A_2 - V_1 \rho V_1 A_1$$

Since $\rho V_2 A_2 = \rho V_1 A_1 = \dot{m} = 157\ kg/s$

$$F = p_1 A_1 - \dot{m}(V_2 - V_1)$$

(+3 points)

Thus,

$$F = (187\ kPa)\left(\frac{\pi}{4}(0.2m)^2\right) - \left(157\frac{kg}{s}\right)\left(\left(20\frac{m}{s}\right) - \left(5\frac{m}{s}\right)\right) = 3520\ N$$

(+2 points)