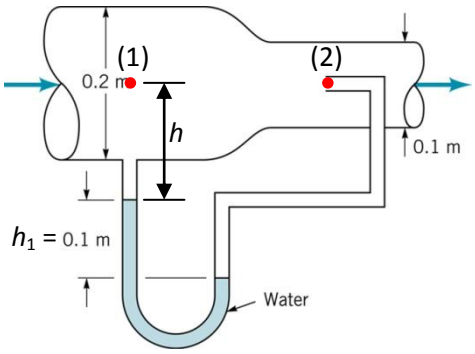
September 24, 2012

NAME

Fluids-ID

Quiz 4. Air flows steadily through the variable area pipe shown at the right. Determine the flow rate Q if viscous and compressibility effects are negligible.



- $\gamma_{H2O} = 9.80 \times 10^3$ N/m^3
- $\gamma_{air} = 12.0$ N/m^3 (Note that $\gamma_{air} << \gamma_{H2O}$)
- Bernoulli equation:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

Solution:

1) Bernoulli equation

Since $z_1 = z_2$ and $V_2 = 0$,

$$\frac{p_1}{\gamma_{air}} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma_{air}} \qquad (1) \qquad \textbf{(+4 points)}$$

2) Manometer

$$p_1 + \gamma_{air} \cdot h + \gamma_{H_2O} \cdot h_1 = p_2 + \gamma_{air}(h + h_1)$$

$\gamma_{H_2O} \gg \gamma_{air}$,

$$\frac{p_2}{\gamma_{air}} = \frac{p_1}{\gamma_{air}} + \left(\frac{\gamma_{H_2O}}{\gamma_{air}}\right) h_1 \qquad (2) \qquad \textbf{(+3 points)}$$

3) Flow rate

$$Q = V_1 A_1$$

where V_1 is from (1) and (2),

$$V_1 = \sqrt{2g\left(\frac{\gamma_{H_2O}}{\gamma_{air}}\right)h} = \sqrt{2\left(9.81\frac{m}{s^2}\right)\left(\frac{9.80\times10^3 N/m^3}{12.0 N/m^3}\right)(0.1m)} = 40.0\frac{m}{s}$$

Thus,

$$Q = \left(40.0\frac{m}{s}\right)\left(\frac{\pi}{4}\right)(0.2m)^2 = 1.26\frac{m^3}{s} \qquad \textbf{(+3 points)}$$