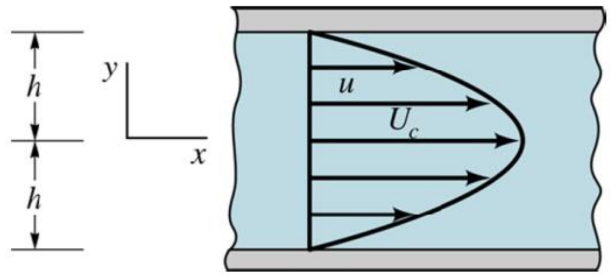
October 31, 2012

NAME

Fluids-ID

Quiz 10. Oil (μ = 0.4 N·s/m^2) flows between two fixed horizontal infinite parallel plates with a spacing of 5 mm. The flow is laminar and steady with a constant pressure gradient dp/dx = -900 N/m^3. Determine the shear stress $\tau_{xy} = \mu(\partial u/\partial y + \partial v/\partial x)$ at $y = h$, by solving Navier Stokes equation.



Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Navier Stokes: $$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{dp}{dx} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

Solution:

Since the flow is steady and parallel, $\partial u/\partial t$ = 0 and v = 0. From the continuity equation, $\partial u/\partial x = 0$. Then, the Navier Stokes equation is rewritten as

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\left(\frac{dp}{dx}\right)$$ (+ 4 points)

By integrating the Navier Stoke equation twice to yield

$$u = \frac{1}{2\mu}\left(\frac{dp}{dx}\right)y^2 + c_1 y + c_2$$

To satisfy the no-slip boundary conditions, i.e. u = 0 at $y = \pm h$,

$$c_1 = 0$$
$$c_2 = -\frac{1}{2\mu}\left(\frac{dp}{dx}\right)h^2$$

Thus, the velocity distribution becomes

$$u = \frac{1}{2\mu}\left(\frac{dp}{dx}\right)(y^2 - h^2)$$ (+4 points)

Hence,

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \left(\frac{dp}{dx}\right)y$$

At $y = h$,

$$\tau_{xy} = \left(-900\,\frac{N}{m^3}\right)\left(\frac{0.005\ m}{2}\right) = -2.25\ N/m^2$$

(+2 points)