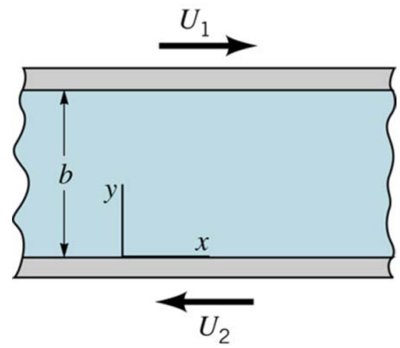
November 1, 2010

NAME

Fluids-ID

Quiz 8. An incompressible, viscous fluid is placed between horizontal, infinite, parallel plates as is shown in the figure at the right. The two plates move in opposite directions with constant velocities, U_1 and U_2, as shown. The pressure gradient in the x direction is zero and the only body force is due to the fluid weight. Use the Navier-Stokes equations to derive an expression for the velocity distribution between the plates. Assume the flow is steady, laminar, and parallel to the plates.



- Navier-Stokes equations:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

Solution:

For this geometry there is no velocity in the y or z direction, i.e., $v = w = 0$, as the flow is parallel to the plates. In this case, $\partial u/\partial x = 0$, from the continuity equation. Furthermore, $\partial u/\partial z = 0$, for infinite plates, and $\partial u/\partial t = 0$ for steady flow. With these conditions and $\partial p/\partial x = 0$, the Navier-Stokes equation reduces to

$$\frac{\partial^2 u}{\partial y^2} = 0 \qquad \text{(+5 points)}$$

By integrating the above equation,

$$u = C_1 y + C_2 \qquad \text{(+2 points)}$$

For y = 0, $u = -U_2$, so that

$$C_2 = -U_2$$

For y = b, $u = U_1$, so that

$$C_1 = \frac{U_1 + U_2}{b}$$

Thus,

$$u = \left(\frac{U_1 + U_2}{b}\right) y - U_2 \qquad \text{(+3 points)}$$