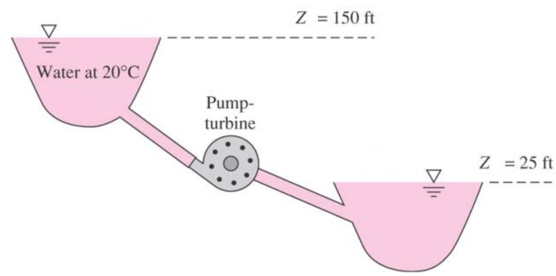
October 27, 2010

NAME

Fluids-ID

Quiz 7. The *pump-turbine* system in the figure draws water from the upper reservoir in the daytime to produce power for a city. At night, it pumps water from lower to upper reservoirs to restore the situation. For a design flow rate of 15,000 gal/min (or 33.4 ft^3/s) in either direction, the frictional head loss is 17 ft. Estimate the power in horse power (a) extracted by the turbine and (b) delivered by the pump. (Note: $\dot{W}_t = \gamma Q h_t$ and $\dot{W}_p = \gamma Q h_p$; γ = 62.4 lbf/ft^3; 1 hp = 550 ft·lbf/s)



- Energy equation:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Solution:

(a) For turbine operation, take the upper reservoir surface as the point "1" (i.e., upstream) in the energy equation, and with $p_1 = p_2 = 0$, $V_1 \approx V_2 \approx 0$, and $h_p = 0$,

$$0 + 0 + z_1 + 0 = 0 + 0 + z_2 + h_t + h_L$$

$$\therefore h_t = z_1 - z_2 - h_L = 150\ \text{ft} - 25\ \text{ft} - 17\ \text{ft} = 108\ \text{ft}$$

(+3 points)

Thus,

$$\dot{W}_t = \gamma Q h_t = \left(62.4 \frac{\text{lbf}}{\text{ft}^3}\right)\left(33.4 \frac{\text{ft}^3}{\text{s}}\right)(108\ \text{ft}) = 225{,}000\ \frac{\text{lbf} \cdot \text{ft}}{\text{s}} = \mathbf{410\ hp}$$

(+2 points)

(b) For pump operation, take the lower reservoir surface as the point "1" (i.e., upstream) in the energy equation, and with $p_1 = p_2 = 0$, $V_1 \approx V_2 \approx 0$, and $h_t = 0$,

$$0 + 0 + z_1 + h_p = 0 + 0 + z_2 + h_L$$

(+3 points)

$$\therefore h_p = z_2 - z_1 + h_L = 150\ \text{ft} - 25\ \text{ft} + 17\ \text{ft} = 142\ \text{ft}$$

Thus,

$$\dot{W}_p = \gamma Q h_p = \left(62.4 \frac{\text{lbf}}{\text{ft}^3}\right)\left(33.4 \frac{\text{ft}^3}{\text{s}}\right)(142\ \text{ft}) = 296{,}000\ \frac{\text{lbf} \cdot \text{ft}}{\text{s}} = \mathbf{540\ hp}$$

(+2 points)