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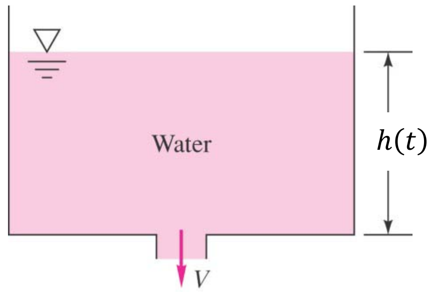
NAME

Fluids-ID

Quiz 5. The cylindrical container shown in the figure is *D* = 20 cm in diameter and has an exit hole of *d* = 3 cm diameter at the bottom. The tank contains fresh water. If the water surface is falling at the nearly steady rate $dh/dt \approx -0.072$ m/s, estimate the average velocity *V* from the bottom exit



- Conservation of mass:

$$\frac{d}{dt}\int_{CV} \rho dV + \sum \dot{m}_{out} - \sum \dot{m}_{in} = 0$$

Solution:

From the conservation of mass law with one outlet and no inlet,

$$\frac{d}{dt}\int_{CV} \rho dV + \dot{m}_{out} = 0 \qquad \text{(+3 points)}$$

By noting that $\int_{CV} \rho dV = \rho \int_{CV} dV = \rho \forall$ is the mass of the water in the tank at time t and $\dot{m}_{out} = \rho A V$ is the mass flow rate through the exit hole,

$$\frac{d}{dt}\left(\rho \frac{\pi D^2}{4} h(t)\right) + \rho \frac{\pi d^2}{4} V = 0$$

or

$$\rho \frac{\pi D^2}{4} \frac{dh}{dt} + \rho \frac{\pi d^2}{4} V = 0 \qquad \text{(+5 points)}$$

Thus,

$$V = -\left(\frac{D}{d}\right)^2 \frac{dh}{dt} = -\left(\frac{20\ cm}{3\ cm}\right)^2 \left(-0.072 \frac{m}{s}\right) = \mathbf{3.2} \frac{\boldsymbol{m}}{\boldsymbol{s}} \qquad \text{(+2 points)}$$