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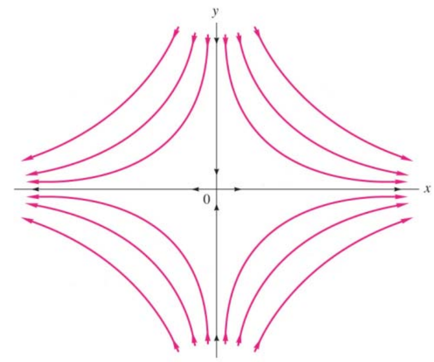
NAME

Fluids-ID

Quiz 4. The velocity field near a stagnation point (see figure) may be written in the form



$$u = \frac{U_0 x}{L} \qquad v = -\frac{U_0 y}{L}$$

where U_0 and L are constants. For the particular case L = 1.5 m, if the resultant acceleration at (x, y) = (1 m, 1 m) is 25 m/s^2, what is the value of U_0?

- For two-dimensional flow:

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

<u>Solution:</u>

For two-dimensional flow, the acceleration components are

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = 0 + \left(\frac{U_0 x}{L}\right)\left(\frac{U_0}{L}\right) + \left(-\frac{U_0 y}{L}\right)(0) = \frac{U_0^2}{L^2}x \qquad \text{(+4 points)}$$

and

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = 0 + \left(\frac{U_0 x}{L}\right)(0) + \left(-\frac{U_0 y}{L}\right)\left(-\frac{U_0}{L}\right) = \frac{U_0^2}{L^2}y \qquad \text{(+4 points)}$$

For the given resultant acceleration of 25 m/s2 at (x, y) = (1 m, 1 m),

$$|a| = 25\frac{m}{s^2} = \sqrt{a_x^2 + a_y^2} = \frac{U_0^2}{L^2}\sqrt{x^2 + y^2} = \frac{U_0^2}{(1.5\,m)^2}\sqrt{2\,m}$$

Solve for U_0,

$$U_0 = 6.3\frac{m}{s} \qquad \text{(+2 points)}$$