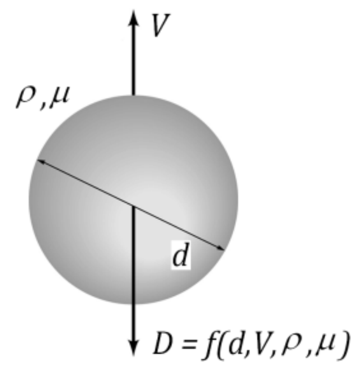
November 6, 2009

NAME

Fluids-ID

Quiz 9.

The drag, D, on a sphere moving in a fluid can be expressed as $D = f(d,V,\rho,\mu)$ where d is the spear diameter, V is the sphere velocity, ρ and μ are respectively the density and viscosity of the fluid. (a) Develop a suitable set of pi terms by using the d, V, and ρ as the repeating variables. (b) Drag D = 10 N for a sphere, with a diameter d = 5 cm, moving at V = 4 m/s in water. For a balloon with d = 1 m diameter rising in air, determine the velocity V and the drag D, if the pi terms in (a) are same for both the sphere and the balloon.
(For water, ρ = 999 kg/m^3 and $\mu = 1.12 \times 10^{-3}$ N·s/m^2; For air, ρ = 1.23 kg/m^3 and $\mu = 1.79 \times 10^{-5}$ N·s/m^2)



Solution:

(a) From the pi theorem, 5 -3 = 2 pi terms required.

$$\Pi_1 = \mu \cdot \rho^a \cdot V^b \cdot d^c$$
$$(ML^{-1}T^{-1})(ML^{-3})^a(LT^{-1})^b(L)^c = M^0L^0T^0$$
$$\therefore \Pi_1 = \frac{\mu}{\rho V d}$$

(+3 points)

$$\Pi_2 = D \cdot \rho^a \cdot V^b \cdot d^c$$
$$(MLT^{-2})(ML^{-3})^a(LT^{-1})^b(L)^c = M^0L^0T^0$$
$$\therefore \Pi_2 = \frac{D}{\rho V^2 d^2}$$

(+3 points)

(b) Let S: sphere and B: balloon
For Π_1,

$$\left(\frac{\mu}{\rho V d}\right)_S = \left(\frac{\mu}{\rho V d}\right)_B$$
$$V_B = \left(\frac{\rho_S}{\rho_B}\right)\left(\frac{\mu_B}{\mu_S}\right)\left(\frac{d_S}{d_B}\right)V_S$$

$$= \left(\frac{999\ \mathrm{kg/m^3}}{1.23\ \mathrm{kg/m^3}}\right)\left(\frac{1.79\times10^{-5}\ \mathrm{N\cdot s/m^2}}{1.12\times10^{-3}\ \mathrm{N\cdot s/m^2}}\right)\left(\frac{0.05\ \mathrm{m}}{1\ \mathrm{m}}\right)\left(4\,\frac{\mathrm{m}}{\mathrm{s}}\right) = 2.6\,\frac{\mathrm{m}}{\mathrm{s}}$$ (+2 points)

For Π_2,

$$\left(\frac{D}{\rho V^2 d^2}\right)_S = \left(\frac{D}{\rho V^2 d^2}\right)_B$$

$$D_B = \left(\frac{\rho_B}{\rho_S}\right)\left(\frac{V_B}{V_S}\right)^2\left(\frac{d_B}{d_S}\right)^2 D_S$$

$$= \left(\frac{1.23\ \mathrm{kg/m^3}}{999\ \mathrm{kg/m^3}}\right)\left(\frac{2.6\ \mathrm{m/s}}{4\ \mathrm{m/s}}\right)^2\left(\frac{1\ \mathrm{m}}{0.05\ \mathrm{m}}\right)^2 (10\ \mathrm{N}) = 2.1\ \mathrm{N}$$ (+2 points)