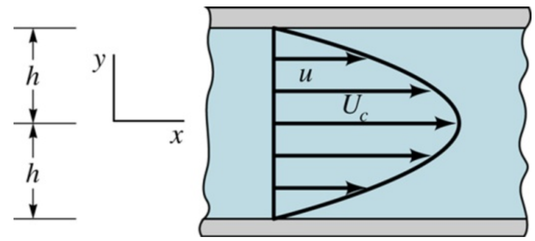
November 2, 2009

NAME

Fluids-ID

Quiz 8.

Oil (μ = 0.4 N·s/m^2) flows between two fixed horizontal infinite parallel plates with a spacing of $2 \cdot h$ = 5 mm. The flow is laminar and steady with a constant pressure gradient dp/dx = -900 N/m^3. Start with the Navier Stokes equation and determine the flow rate q passing between the plates (for a unit width).



Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Navier Stokes: $$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{dp}{dx} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

<u>Solution:</u>

Since the flow is steady, $\partial u/\partial t$ = 0 and as the flow is parallel, v = 0, thus $\partial u/\partial x$ = 0 from the continuity equation. Then, the Navier Stokes equation is rewritten as

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\left(\frac{dp}{dx}\right)$$ (+ 4 points)

By integrating the Navier Stoke equation twice to yield

$$u = \frac{1}{2\mu}\left(\frac{dp}{dx}\right)y^2 + c_1 y + c_2$$

To satisfy the no-slip boundary conditions, i.e. u = 0 at $y = \pm h$,

$$c_1 = 0$$

$$c_2 = -\frac{1}{2\mu}\left(\frac{dp}{dx}\right)h^2$$

Thus, the velocity distribution becomes

$$u = \frac{1}{2\mu}\left(\frac{dp}{dx}\right)(y^2 - h^2)$$ (+2 points)

Let q be the flow rate per unit width,

$$q = \int_{-h}^{h} u(y)dy$$

(+2 points)

By using the velocity distribution from the Navier Stokes equation,

$$q = \int_{-h}^{h} \frac{1}{2\mu}\left(\frac{dp}{dx}\right)(y^2 - h^2)dy$$

$$= -\frac{2}{3\mu}\left(\frac{dp}{dx}\right)h^3$$

For μ = 0.4 N·s/m^2, dp/dx = -900 N/m^3, and h = 0.0025 m,

$$q = -\frac{2}{3(0.4\ \mathrm{N\cdot s/m^2})}(-900\ \mathrm{N/m^3})(0.0025\ \mathrm{m})^3$$

$$= 2.34 \times 10^{-5}\ \frac{\mathrm{m^2}}{\mathrm{s}}$$

(+2 points)