September 30, 2009

NAME

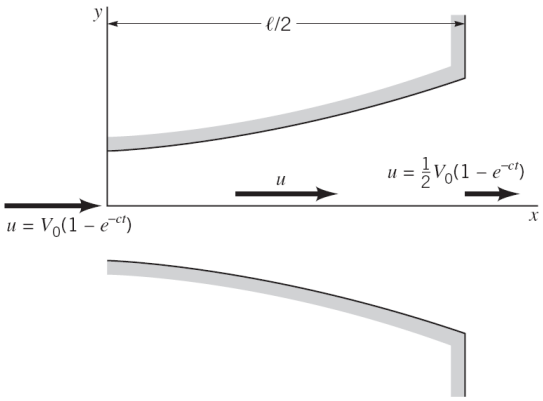
Fluids-ID

Quiz 4.

As a valve is opened, water flows through the diffuser at an increasing flowrate so that the velocity along the centerline is given by

$$\underline{V} = u\hat{\imath} = V_0(1 - e^{-ct})\left(1 - \frac{x}{\ell}\right)\hat{\imath}$$

where V_0, c, and ℓ are constants. If V_0 = 10 ft/s, c = 0.5 1/s and ℓ = 5 ft, determine the acceleration at $x = \ell/2$ = 2.5 ft and t = 2 s.



- Acceleration: $\underline{a} = a_x\hat{\imath} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)\hat{\imath}$

Solution:

With $u = V_0(1 - e^{-ct})\left(1 - \frac{x}{\ell}\right)$, $v = 0$, and $w = 0$,

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$$ (+5 points)

where,

$$\frac{\partial u}{\partial t} = V_0\left(1 - \frac{x}{\ell}\right)c \cdot e^{-ct}$$
$$\frac{\partial u}{\partial x} = V_0(1 - e^{-ct})\left(-\frac{1}{\ell}\right)$$

Thus,

$$a_x = V_0\left(1 - \frac{x}{\ell}\right)\left[ce^{-ct} - \frac{V_0}{\ell}(1 - e^{-ct})^2\right]$$ (+3 points)

At x = 2.5 ft and t = 2 s,

$$a_x = \left(10\frac{\text{ft}}{\text{s}}\right)\left(1 - \frac{2.5\text{ ft}}{5\text{ ft}}\right)\left[0.5e^{-\frac{0.5}{\text{s}}\times 2\text{ s}} - \frac{10\text{ ft/s}}{5\text{ ft}}\left(1 - e^{-\frac{0.5}{\text{s}}\times 2\text{ s}}\right)^2\right] = -3.08\,\text{ft/s}^2$$

(+2 points)