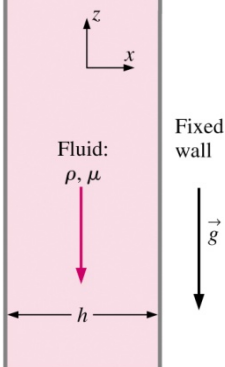


NAME _____

Fluids-ID _____

Quiz 8. Consider steady, incompressible, parallel, laminar flow of a viscous fluid falling between two infinite vertical walls. The distance between the walls is h , and gravity acts in the negative z -direction (downward in the figure). There is no applied (forced) pressure ($\nabla p = 0$) driving the flow – the fluid falls by gravity alone ($g_z = -g$). Calculate the centerline velocity (along $x = 0$ line) if the fluid is glycerin at 20°C and $h = 2$ mm. Assume the flow is purely two-dimensional ($v = 0$, $\partial/\partial y = 0$) and parallel to the walls ($u = 0$).

(For glycerin @ 20°C , $\rho = 1,260$ kg/m³, $\mu = 1.49$ kg/m·s)



Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

z-momentum:

$$\rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$$

Solution

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \text{or} \quad \frac{\partial w}{\partial z} = 0 \quad \because u = v = 0$$

Since $\frac{\partial w}{\partial t} = 0$, $\frac{\partial w}{\partial y} = 0$, $\frac{\partial w}{\partial z} = 0 \Rightarrow w = w(x)$ only.

z-momentum:

$$\rho \left(\underbrace{\frac{\partial w}{\partial t}}_{\text{steady}} + \underbrace{u \frac{\partial w}{\partial x}}_{u=0} + \underbrace{v \frac{\partial w}{\partial y}}_{\partial/\partial y=0} + \underbrace{w \frac{\partial w}{\partial z}}_{\text{continuity}} \right) = \underbrace{-\frac{\partial p}{\partial z}}_{\nabla p=0} + \underbrace{\rho g_z}_{-\rho g} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \underbrace{\frac{\partial^2 w}{\partial y^2}}_{\partial/\partial y=0} + \underbrace{\frac{\partial^2 w}{\partial z^2}}_{\text{continuity}} \right)$$

or

$$\frac{d^2 w}{dx^2} = \frac{\rho g}{\mu}$$

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Integration of z-momentum equation gives,

$$w = \frac{\rho g}{2\mu} x^2 + C_1 x + C_2$$

B.C.

$$w(x = h/2) = 0:$$

$$0 = \frac{\rho g}{8\mu} h^2 + C_1 \frac{h}{2} + C_2$$

$$w(x = -h/2) = 0$$

$$0 = \frac{\rho g}{8\mu} h^2 - C_1 \frac{h}{2} + C_2$$

$$\Rightarrow C_1 = 0, C_2 = -\frac{\rho g}{8\mu} h^2$$

Thus,

$$w = \frac{\rho g}{2\mu} \left(x^2 - \left(\frac{h}{2} \right)^2 \right)$$

Centerline velocity:

$$w(x = 0) = \frac{\rho g}{2\mu} \left(- \left(\frac{h}{2} \right)^2 \right) = -\frac{\rho g h^2}{8\mu}$$

or

$$w = -\frac{1260 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2 \times (0.002 \text{ m})^2}{8 \times 1.49 \text{ kg/m} \cdot \text{s}} = -4.15 \text{ mm/s}$$