October 24, 2008

NAME

Fluids-ID

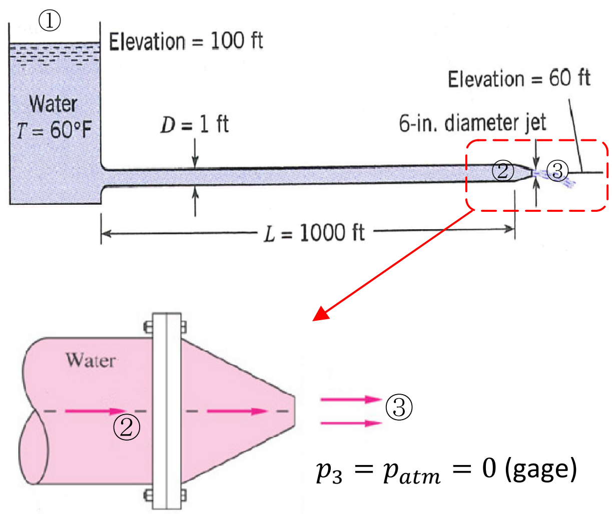
Quiz 7. Water flows from a large reservoir through a pipe and then discharges from a nozzle as shown below.

a) What is the discharge of water Q? The head loss in the pipe itself is given as $h_L = 0.02(L/D)(V^2/2g)$, where L and D are the length and diameter of the pipe and V is the velocity in the pipe. (Hint: Use the energy eq. between 1 and 3 and continuity between 2 and 3)

b) What is the gage pressure p_2 at point (2)? (Hint: Assume frictionless flow and use the Bernoulli eq. between 2 and 3)

c) Find the force F_B exerted by the flange bolts to hold the nozzle. (Use the momentum eq.)

(Water density ρ is 1.94 slug/ft^3 and gravity g is 32.2 ft/s^2 at 60°F)



Continuity equation:

$$\sum \dot{m}_{out} - \sum \dot{m}_{in} = 0$$

Momentum equation:

$$\sum \underline{F} = \sum(\dot{m}\underline{V})_{out} - \sum(\dot{m}\underline{V})_{in}$$

Energy equation:

$$\frac{p_i}{\gamma} + \frac{V_i^2}{2g} + z_i + h_p = \frac{p_o}{\gamma} + \frac{V_o^2}{2g} + z_o + h_t + h_L$$

<u>Solution:</u>

(a) <u>Energy equation between ①and ③</u>:

$$\cancel{\frac{p_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + z_1 + \cancel{h_p} = \cancel{\frac{p_3}{\gamma}} + \frac{V_3^2}{2g} + z_3 + \cancel{h_t} + h_L \qquad (1)$$

where, $p_1 = p_3 = p_{atm}$, $V_1 \approx 0$, $h_p = h_t = 0$, and head loss h_L is

$$h_L = 0.02\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) = 0.02\left(\frac{L}{D}\right)\left(\frac{V_2^2}{2g}\right) \qquad (2) \qquad \because V = V_2 \text{ through the pipe due to continuity}$$

V_2 is from the continuity between ② and ③ (let d = jet diameter)

$$V_2 A_2 = V_3 A_3 \quad \therefore V_2 = V_3 \frac{A_3}{A_2} = V_3 \left(\frac{d}{D}\right)^2 \qquad (3)$$

Then, plugging (2)+(3) into (1) gives

$$V_3 = \sqrt{\frac{2g(z_1 - z_3)}{1 + 0.02\left(\frac{L}{D}\right)\left(\frac{d}{D}\right)^4}} = 33.8\ \text{ft/s}$$

$$\therefore Q = V_3 A_3 = V_3\left(\frac{\pi}{4}d^2\right) = \mathbf{6.64\ ft^3/s}$$

(b) Bernoulli equation between ② and ③:

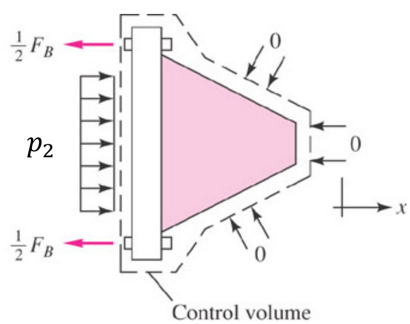
$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + \cancel{z_2} = \cancel{\frac{p_3}{\gamma}} + \frac{V_3^2}{2g} + \cancel{z_3}$$

where, p_3 = 0 gage and $z_2 = z_3$.

$$\therefore p_2 = \frac{\gamma}{2g}\left(V_3^2 - V_2^2\right) = \frac{\rho}{2}V_3^2\left(1 - \left(\frac{d}{D}\right)^2\right) = \mathbf{831\ lbf/ft^2\ gage}$$

(c) Momentum equation:

For a control volume which encloses the nozzle as shown below,



$$\sum F_x = \dot{m}V_3 - \dot{m}V_2$$

The control volume force valance shows that

$$\sum F_x = -F_B + p_2 A_2$$

Then,

$$-F_B + p_2 A_2 = \dot{m}(V_3 - V_2)$$

$$\therefore F_B = p_2\left(\frac{\pi}{4}D^2\right) - \rho Q V_3\left(1 - \left(\frac{d}{D}\right)^2\right) = \mathbf{326\ lbf}$$