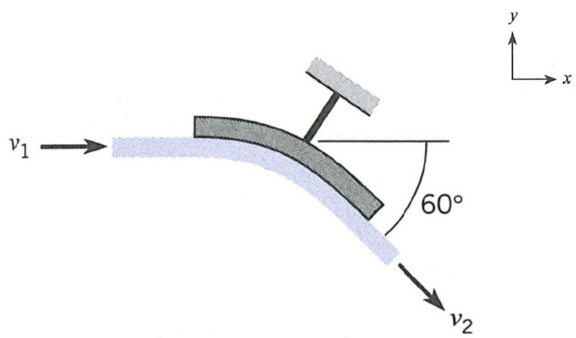
NAME

Fluids-ID

Quiz 6. A water jet is deflected 60° by a stationary vane as shown in the figure. The incoming jet has a speed of 100 ft/s and a diameter of 1 in. Find the ***force exerted by the jet on the vane***. Neglect the influence of gravity and assume irrotational flow. (Water density ρ = 1.94 slug/ft^3)



Momentum equation:

$$\sum \underline{\boldsymbol{F}} = \frac{\partial}{\partial t}\int_{CV} \underline{\boldsymbol{V}} \rho dV\!\!\!/ + \int_{CS} \underline{\boldsymbol{V}} \rho \underline{\boldsymbol{V}} \cdot d\underline{\boldsymbol{A}}$$

1) Steady flow:

$$\frac{\partial}{\partial t}\int_{CV} \underline{\boldsymbol{V}} \rho dV\!\!\!/ = 0$$

2) Uniform flow across A:

$$\int_{CS} \underline{\boldsymbol{V}} \rho \underline{\boldsymbol{V}} \cdot d\underline{\boldsymbol{A}} = \sum_{CS} \underline{\boldsymbol{V}}(\rho \underline{\boldsymbol{V}} \cdot \underline{\boldsymbol{A}})$$

Solution:

Bernoulli equation: $\cancel{p_1} + \frac{1}{2}\rho v_1^2 + \gamma \cancel{z_1} = \cancel{p_2} + \frac{1}{2}\rho v_2^2 + \gamma \cancel{z_2}$ $\quad \therefore v_1 = v_2 = v$

Continuity equation: $v_1 A_1 = v_2 A_2 \quad \therefore A_1 = A_2 = A$

x-Momentum equation:

$$F_x = v_1(-\rho v_1 A_1) + v_2 \cos 60° \, (\rho v_2 A_2) = -\rho A v^2 (1 - \cos 60°)$$

$$= -\left(1.94 \frac{\text{slug}}{\text{ft}^3}\right) \frac{\pi}{4} \left(\frac{1}{12}\text{ft}\right)^2 \left(100 \frac{\text{ft}}{\text{s}}\right)^2 (1 - \cos 60°) = -53.0 \text{ lbf}$$

y-Momentum equation:

$$F_y = -v_2 \sin 60° \, (\rho v_2 A_2) = -\rho A v^2 \sin 60°$$

$$= -\left(1.94 \frac{\text{slug}}{\text{ft}^3}\right) \frac{\pi}{4} \left(\frac{1}{12}\text{ft}\right)^2 \left(100 \frac{\text{ft}}{\text{s}}\right)^2 (\sin 60°) = -91.8 \text{ lbf}$$

The force of the jet on the vane is opposite in direction to the force required to hold the vane stationary. Therefore,

$$\underline{\boldsymbol{F}}_{jet} = (53.0 \text{ lbf})\hat{\boldsymbol{\imath}} + (91.8 \text{ lbf})\hat{\boldsymbol{\jmath}}$$