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NAME

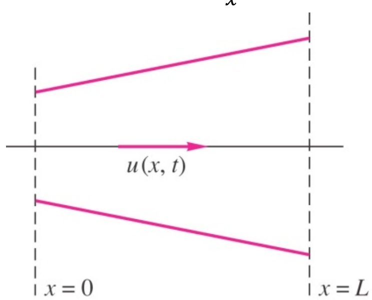
Fluids-ID

Quiz 4. When a valve is opened, fluid flows in the expansion duct shown below according to the approximation

$$\underline{V} = u\hat{\boldsymbol{\imath}} = U\left(1-\frac{x}{2L}\right)\left(\frac{Ut}{L}\right)\hat{\boldsymbol{\imath}}$$

for t $\ll L/U$. If L = 1 m and U = 1 m/s, then at (x, t) = $(L, L/2U)$,

1) Find the unsteady (local) acceleration of a_x
2) Find the convective acceleration of a_x
3) Find the total acceleration a_x



Acceleration:

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

Solution:

1) local acceleration

$$(a_x)_{local} = \frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\left[U\left(1-\frac{x}{2L}\right)\left(\frac{Ut}{L}\right)\right]$$

$$= \frac{U^2}{L}\left(1-\frac{x}{2L}\right)$$

at (x, t) = $(L, L/2U)$,

$$(a_x)_{local} = \frac{U^2}{L}\left(1-\frac{L}{2L}\right) = \frac{U^2}{2L} = \frac{(1\ \mathrm{m/s})^2}{2\times 1\ \mathrm{m}} = \mathbf{0.5}\,\frac{\mathbf{m}}{\mathbf{s^2}}$$

2) convective acceleration

$$(a_x)_{conv} = u\frac{\partial u}{\partial x} = U\left(1-\frac{x}{2L}\right)\left(\frac{Ut}{L}\right)\frac{\partial}{\partial x}\left[U\left(1-\frac{x}{2L}\right)\left(\frac{Ut}{L}\right)\right]$$

$$= -\frac{U^2}{L}\left(1-\frac{x}{2L}\right)\left(\frac{Ut}{L}\right)^2$$

at (x, t) = $(L, L/2U)$,

$$(a_x)_{conv} = -\frac{U^2}{L}\left(1-\frac{L}{2L}\right)\left(\frac{U}{L}\cdot\frac{L}{2U}\right)^2 = -\frac{U^2}{8L} = -\frac{(1\ \mathrm{m/s})^2}{8\times 1\ \mathrm{m}} = \mathbf{-0.125}\,\frac{\mathbf{m}}{\mathbf{s^2}}$$

3) total acceleration

$$a_x = (a_x)_{local} + (a_x)_{conv} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$$

$$= \frac{U^2}{2L}\left(1 - \frac{x}{2L}\right)\left(1 - \left(\frac{Ut}{L}\right)^2\right)$$

at (x, t) = $(L, L/2U)$,

$$a_x = \frac{U^2}{L}\left(1 - \frac{L}{2L}\right)\left(1 - \left(\frac{U}{L}\cdot\frac{L}{2U}\right)^2\right) = \frac{3U^2}{8L} = \frac{3\times(1\ \mathrm{m/s})^2}{8\times 1\ \mathrm{m}} = 0.375\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

or $$a_x = (a_x)_{local} + (a_x)_{conv} = 0.5\,\frac{\mathrm{m}}{\mathrm{s}^2} - 0.125\,\frac{\mathrm{m}}{\mathrm{s}^2} = \mathbf{0.375\,\frac{m}{s^2}}$$