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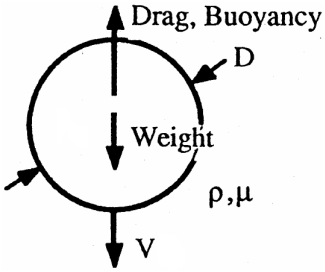
NAME

Fluids-ID

Quiz 11. A sphere weights 250 g and is 7.35 cm in diameter. It is dropped from rest from the surface of a 100-m-depth lake. Assuming a laminar-flow drag coefficient C_D = 0.5, estimate (a) its falling velocity and (b) the time to reach to the lake bottom. Neglect the initial transient state and assume that the sphere falls at constant velocity, known as the 'terminal velocity', from the beginning. (ρ = 1000 kg/m^3, $\nu = 1.12 \times 10^{-6}$ m^2/s)



For a sphere,

$$\text{Buoyancy} = \left(\frac{4}{3}\pi R^3\right)\rho g$$

$$\text{Drag} = \frac{1}{2}\rho C_D V^2(\pi R^2)$$

where R is the radius.

Solution:

(a) Terminal velocity

$$\text{Weight} = \text{Buoyancy} + \text{Drag}$$

or

$$mg = \left(\frac{4}{3}\pi R^3\right)\rho g + \frac{1}{2}\rho C_D V^2(\pi R^2)$$

Thus,

$$V = \sqrt{\frac{8g}{\pi\rho C_D D^2}\left(m - \frac{\pi\rho D^3}{6}\right)}$$

$$= \sqrt{\frac{(8)(9.81)}{\pi(1000)(0.5)(0.0735)^2}\left(0.25 - \frac{\pi(1000)(0.0735)^3}{6}\right)} = \mathbf{0.624\ m/s}$$

Re check:

$$Re = \frac{VD}{\nu} = \frac{(0.624)(0.0735)}{1.21\times10^{-5}} = 4.1 \times 10^4 \qquad (C_D \approx 0.5 \text{ from Fig. 9.21 in pp. 526})$$

(b) Falling time

$$t = \frac{h}{V} = \frac{100}{0.624} = 160\text{ s} = \mathbf{2\ m\ 40\ s}$$