December 10, 2007

NAME

Fluids-ID

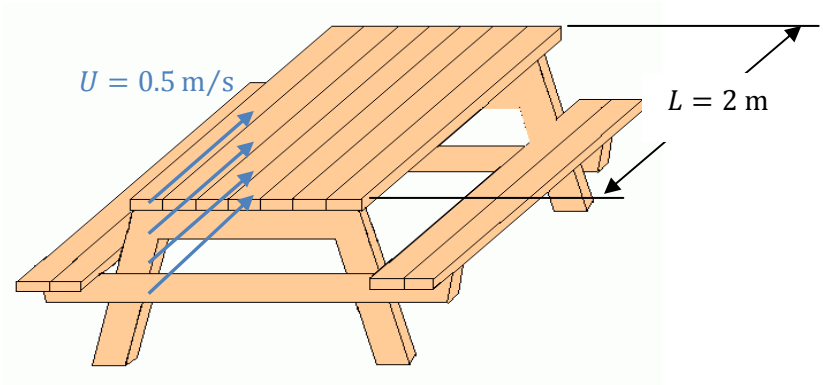
Quiz 9. During a warm and sunny day in Iowa City, wind passes over a picnic table at City Park. Air passes directly over the 2 m long table. Determine the boundary layer thickness and the wall shear stress at the trailing edge of the table for a velocity of 0.5 m/s. Is boundary layer thickness maximum at the trailing edge? Assume that the table is perfectly flat and smooth, and has a sharp leading edge.

$Re_{crit} = 2 \times 10^5 \sim 3 \times 10^6$

$\rho = 1.23\,\mathrm{kg/m^3}$

$\nu = 1.46 \times 10^{-5}\ \mathrm{m^2/s}$

$\mu = \rho\nu = 1.79 \times 10^{-5}\,\mathrm{N \cdot s/m^2}$



Reynolds number: $Re_L = \frac{UL}{\nu}$

Laminar flow:

$$\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}}$$

$$\tau_w = 0.332U^{3/2}\sqrt{\frac{\rho\mu}{x}}$$

Turbulent flow:

$$\frac{\delta}{x} = \frac{0.370}{Re^{1/5}}$$

$$\tau_w = 0.0225\rho U^2\left(\frac{\nu}{U\delta}\right)^{1/4}$$

Solution:

$Re_L = \frac{UL}{\nu} = \frac{0.5\mathrm{m/s} \times 2\ \mathrm{m}}{1.46\times10^{-5}\mathrm{m^2/s}} = 6.85 \times 10^4 \Rightarrow$ Laminar flow (+3 points)

At $x = L$,

$$\delta = \left.\frac{5x}{\sqrt{Ux/\nu}}\right|_{x=L} = \frac{5L}{\sqrt{Re_L}} = \frac{5\times2\ \mathrm{m}}{\sqrt{6.85\times10^4}} = \mathbf{0.0382\ m = 3.82\ cm}$$ (+3 points)

$$\tau_w = 0.332U^{3/2}\sqrt{\frac{\rho\mu}{L}}$$

$$= 0.332 \times (0.5\,\mathrm{m/s})^{3/2} \times \sqrt{\frac{1.23\,\mathrm{kg/m^3} \times 1.79 \times 10^{-5}\,\mathrm{N \cdot s/m^2}}{2\ \mathrm{m}}}$$

$$= \mathbf{3.895 \times 10^{-4}\ N/m^2}$$ (+3 points)

Since $\delta \sim x^{1/2}$, maximum thickness occurs at the trailing edge (+1 point)