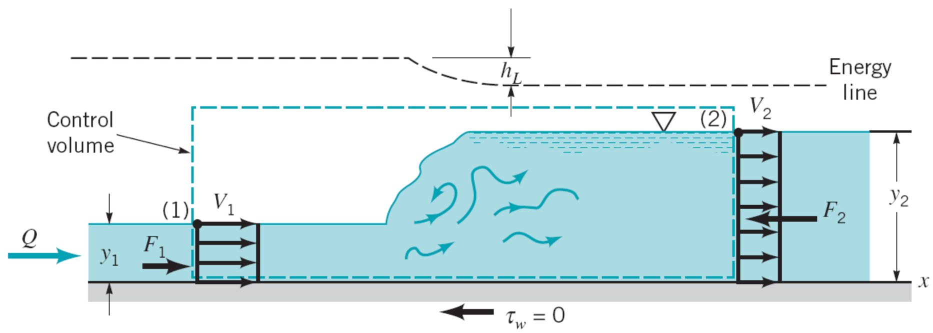
NAME

Fluids-ID

Quiz 6. A high-speed channel flow (V_1, y_1) may "jump" to a low-speed, low-energy condition (V_2, y_2). The pressure at sections 1 and 2 is approximately hydrostatic, and wall friction is negligible. Find (a) V_1 and V_2 using the continuity and momentum relations and (b) head loss h_L using energy equation if y_1 = 0.2 m, y_2 = 1.0 m, and channel width b = 10m. ($\rho_{water} = 998\,\mathrm{kg/m^3}$)



Solution:

(a) x-momentum equation:

$$\bar{p}_1 A_1 - \bar{p}_2 A_2 = V_1(-\rho V_1 A_1) + V_2(\rho V_2 A_2)$$

Continuity equation: $V_2 = \frac{A_1}{A_2} V_1 = \frac{y_1 b}{y_2 b} V_1 = \frac{y_1}{y_2} V_1$

Then,

$$\left(\gamma \frac{y_1}{2}\right)(b y_1) - \left(\gamma \frac{y_2}{2}\right)(b y_2) = -\rho (b y_1) V_1^2 + \rho (b y_2)\left(\frac{y_1}{y_2} V_1\right)^2$$

$$\frac{\gamma b}{2} y_1^2 \left[1 - \left(\frac{y_2}{y_1}\right)^2\right] = \rho b y_1 \left(\frac{y_1}{y_2} - 1\right) V_1^2$$

$$\frac{\rho g b}{2} y_1^2 \left(1 - \frac{y_2}{y_1}\right)\left(1 + \frac{y_2}{y_1}\right) = \rho b \frac{y_1^2}{y_2}\left(1 - \frac{y_2}{y_1}\right) V_1^2$$

$$V_1^{2\dagger} = \frac{g y_2}{2}\left(1 + \frac{y_2}{y_1}\right)$$

Thus, $V_1 = \sqrt{\frac{g}{2} y_2 \left(1 + \frac{y_2}{y_1}\right)} = \sqrt{\frac{9.81\,\mathrm{m/s^2}}{2} \times 1.0\ \mathrm{m} \times \left(1 + \frac{1.0\ \mathrm{m}}{0.2\ \mathrm{m}}\right)} = \mathbf{5.42\ m/s}$

$V_2 = \frac{y_1}{y_2} V_1 = \frac{0.2\ \mathrm{m}}{1.0\ \mathrm{m}} \times 5.42\ \mathrm{m/s} = \mathbf{1.08\ m/s}$

(b) Energy equation (along free surface):

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + y_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + y_2 + h_L$$

Since $p_1 = p_2 = p_{atm}$

$$\frac{V_1^2}{2g} + y_1 = \frac{V_2^2}{2g} + y_2 + h_L$$

$$\therefore\ {h_L}^{\ddagger} = \frac{1}{2g}(V_1^2 - V_2^2) + (y_1 - y_2)$$

$$= \frac{1}{2 \times 9.81\ \mathrm{m/s^2}} \times (5.42^2 - 1.08^2)\ \mathrm{m^2/s^2} + (0.2 - 1.0)\ \mathrm{m} = \mathbf{0.64\ m}$$

Note:

† Froude number

$$Fn_1^2 = \frac{V_1^2}{gy_1} = \frac{1}{2}\frac{y_2}{y_1}\left(1 + \frac{y_2}{y_1}\right)$$

$$Fn_2^2 = \frac{V_2^2}{gy_2} = \frac{1}{gy_2}\left(\frac{y_1}{y_2}V_1\right)^2 = \left(\frac{y_1}{y_2}\right)^3 \frac{V_1^2}{gy_1} = \left(\frac{y_1}{y_2}\right)^3 \left[\frac{1}{2}\frac{y_2}{y_1}\left(1 + \frac{y_2}{y_1}\right)\right] = \frac{1}{2}\frac{y_1}{y_2}\left(1 + \frac{y_1}{y_2}\right)$$

‡ Head loss

$$h_L = \frac{1}{2g}(V_1^2 - V_2^2) + (y_1 - y_2)$$

$$= \frac{1}{2}\left[y_1 \underbrace{\left(\frac{V_1^2}{gy_1}\right)}_{Fn_1^2} + y_2 \underbrace{\left(\frac{V_2^2}{gy_2}\right)}_{Fn_2^2}\right] + (y_1 - y_2)$$

$$= \frac{1}{2}\left[y_1 \frac{1}{2}\frac{y_2}{y_1}\left(1 + \frac{y_2}{y_1}\right) - y_2 \frac{1}{2}\frac{y_1}{y_2}\left(1 + \frac{y_1}{y_2}\right)\right] + (y_1 - y_2)$$

$$= \frac{1}{4}\left[\frac{y_2}{y_1}(y_1 + y_2) - \frac{y_1}{y_2}(y_2 + y_1)\right] + (y_1 - y_2)$$

$$= \frac{(y_1 + y_2)^2(y_2 - y_1)}{4y_1y_2} + (y_1 - y_2) \qquad \because y_2^2 - y_1^2 = (y_2 + y_1)(y_2 - y_1)$$

$$= \frac{(y_2 - y_1)^3}{4y_1y_2}$$