057:20 **Quiz # 08** **11/01/2006**

If the drag F_D of a sphere in a fluid flowing past the sphere is a function of the viscosity μ, the mass density ρ, the velocity of flow V, and the diameter of the sphere D, what dimensionless parameters are applicable to the flow process?

$$F_D = f\left(\mu, \rho, V, D\right)$$

057:20 Quiz # 08 11/01/2006

example 8.2

If the drag F_D of a sphere in a fluid flowing past the sphere is a function of the viscosity μ, the mass density ρ, the velocity of flow V, and the diameter of the sphere D, what dimensionless parameters are applicable to the flow process?

Solution

$$F_D = f(V, \rho, \mu, D) \tag{8.7}$$

In this problem we have five dimensional parameters; namely, F_D, V, ρ, μ, and D. From the Buckingham Π theorem there should be two π-groups. We approach this problem in the same way as used in Example 8.1, by using a table and eliminating the basic dimensions step-by-step.

Variable	[ ]	Variable	[ ]	Variable	[ ]	Variable	[ ]
F_D	$\frac{ML}{T^2}$	$\frac{F_D}{D}$	$\frac{M}{T^2}$	$\frac{F_D}{\rho D^4}$	$\frac{1}{T^2}$	$\frac{F_D}{\rho V^2 D^2}$	0
V	$\frac{L}{T}$	$\frac{V}{D}$	$\frac{1}{T}$	$\frac{V}{D}$	$\frac{1}{T}$		
ρ	$\frac{M}{L^3}$	ρD^3	M				
μ	$\frac{M}{LT}$	μD	$\frac{M}{T}$	$\frac{\mu}{\rho D^2}$	$\frac{1}{T}$	$\frac{\mu}{\rho VD}$	0
D	L						

We can start by using the sphere diameter D to eliminate the length dimension in the other variables. For example, dividing the force by the diameter

*Note that, in rare instances, the number of parameters may be one more than predicted by the Buckingham Π theorem. This anomaly can occur because it is possible that two-dimensional categories can be eliminated when dividing (or multiplying) by a given variable. See Ipsen (7, page 172) for an example of this.

removes the length dimension. Also, multiplying the density by the diameter cubed removes the length dimension. The resulting variables are shown in the second column with their corresponding dimensions. We can use ρD^3 to eliminate the mass dimension in the other variables, and now only three variables remain with dimensions of time only. The result is shown in the third column. The V/D grouping in the second row did not change because there was no mass dimension in this term. Finally, the V/D grouping is used to eliminate the time dimension resulting in the fourth column. There are two π-groups remaining which leads to an equation in the form:

$$\frac{F_D}{\rho V^2 D^2} = f\left(\frac{\mu}{\rho VD}\right) \tag{8.8}$$

Although Eq. (8.8) still has the same variables as Eq. (8.7), the number of terms has been reduced from five to two. This makes the correlation of data much simpler.

057:20 **Quiz # 08** **11/01/2006**

This problem can also be solved using repeating varialbe method (exponent method). The form of the solution is not unique. There could be other forms of PI groups.