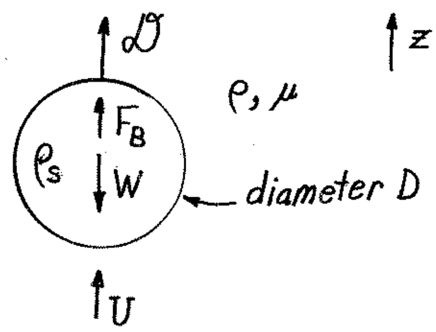
9.45

9.45 A sphere of diameter D and density ρ_s falls at a steady rate through a liquid of density ρ and viscosity μ. If the Reynolds number, $\mathrm{Re} = \rho D U/\mu$, is less than 1, show that the viscosity can be determined from $\mu = gD^2(\rho_s - \rho)/18\,U$.



For steady flow $\sum F_z = 0$

or $\mathscr{D} + F_B = W$, where F_B = buoyant force $= \rho g \forall = \rho g \left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3$

W = weight $= \rho_s g \forall = \rho_s g \left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3$

and $\mathscr{D}$ = drag $= C_D \frac{1}{2}\rho \frac{\pi}{4} D^2$, or since $Re < 1$

$\mathscr{D} = 3\pi D U \mu$

Thus,

$$3\pi D U \mu + \rho g \left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3 = \rho_s g \left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3$$

which can be rearranged to give

$$\mu = \frac{g D^2 (\rho_s - \rho)}{18\, U}$$