

5.46

5.46 Water flows as two free jets from the tee attached to the pipe shown in Fig. P5.46. The exit speed is 15 m/s. If viscous effects and gravity are negligible, determine the  $x$  and  $y$  components of the force that the pipe exerts on the tee.

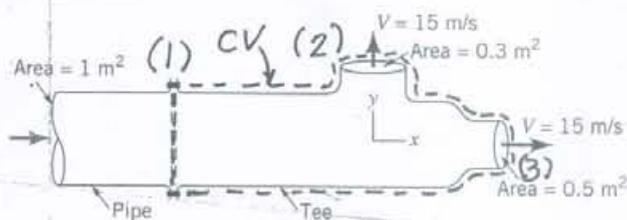


FIGURE P5.46

Use the control volume shown.

For the  $x$ -component of the force exerted by the pipe on the tee we use the  $x$ -component of the linear momentum equation.

$$\begin{aligned} -V_1 \rho V_1 A_1 + V_3 \rho V_3 A_3 &= P_1 A_1 - P_3 A_3 - P_{atm} (A_1 - A_3) + F_x \\ &= (P_{gage} + P_{atm}) A_1 - (P_{gage} + P_{atm}) A_3 - P_{atm} (A_1 - A_3) + F_x \\ &= P_{gage} A_1 + F_x \end{aligned} \quad (1)$$

To get  $V_1$ , we use conservation of mass

$$\begin{aligned} Q_1 &= Q_2 + Q_3 \\ \text{or } A_1 V_1 &= A_2 V_2 + A_3 V_3 \\ \text{so } V_1 &= \frac{A_2 V_2 + A_3 V_3}{A_1} = \frac{(0.3 \text{ m}^2)(15 \text{ m/s}) + (0.5 \text{ m}^2)(15 \text{ m/s})}{1 \text{ m}^2} = 12 \text{ m/s} \end{aligned}$$

To estimate  $P_{gage}$  we use Bernoulli's equation for flow between (1) and (2)

$$\begin{aligned} \frac{P_{gage}}{\rho} + \frac{V_1^2}{2} &= \frac{P_{gage}}{\rho} + \frac{V_2^2}{2} \\ P_{gage} &= \rho \left( \frac{V_2^2 - V_1^2}{2} \right) = \left( 999 \frac{\text{kg}}{\text{m}^3} \right) \left[ \frac{(15 \frac{\text{m}}{\text{s}})^2 - (12 \frac{\text{m}}{\text{s}})^2}{2} \right] \left( \frac{1 \text{ N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} \right) \\ P_{gage} &= 40,500 \frac{\text{N}}{\text{m}^2} \end{aligned}$$

Now using Eq. (1) we get:

$$\begin{aligned} \left[ -(12 \frac{\text{m}}{\text{s}}) \left( 999 \frac{\text{kg}}{\text{m}^3} \right) \left( 12 \frac{\text{m}}{\text{s}} \right) (1 \text{ m}^2) + (15 \frac{\text{m}}{\text{s}}) \left( 999 \frac{\text{kg}}{\text{m}^3} \right) \left( 15 \frac{\text{m}}{\text{s}} \right) (0.5 \text{ m}^2) \right] \left( \frac{1 \text{ N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} \right) &= \\ (40,500 \frac{\text{N}}{\text{m}^2}) (1 \text{ m}^2) + F_x & \end{aligned}$$

$$\text{or } -72,000 \text{ N} = F_x$$

$$\text{so } F_x = \underline{72,000 \text{ N}} \leftarrow$$

For the  $y$  component of the force exerted by the pipe on the tee we use the  $y$  component of the linear momentum equation to get

$$\begin{aligned} V_2 \rho V_2 A_2 &= F_y \\ (15 \frac{\text{m}}{\text{s}}) \left( 999 \frac{\text{kg}}{\text{m}^3} \right) \left( 15 \frac{\text{m}}{\text{s}} \right) (0.3 \text{ m}^2) &= \underline{67,400 \text{ N}} \uparrow = F_y \end{aligned}$$