7.55

7.55 For a certain model study involving a 1:5 scale model it is known that Froude number similarity must be maintained. The possibility of cavitation is also to be investigated, and it is assumed that the cavitation number must be the same for model and prototype. The prototype fluid is water at 30 °C, and the model fluid is water at 70 °C. If the prototype operates at an ambient pressure of 101 kPa (abs), what is the required ambient pressure for the model system?

For Froude number similarity,

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}}$$

so that (with $g = g_m$)

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}} \qquad (1)$$

For cavitation number similarity,

$$\frac{(p_r - p_v)_m}{\frac{1}{2}\rho_m V_m^2} = \frac{(p_r - p_v)}{\frac{1}{2}\rho V^2}$$

It follows that

$$(p_r - p_v)_m = \frac{\rho_m}{\rho} \frac{V_m^2}{V^2} (p_r - p_v)$$

and making use of Eq. (1)

$$(p_r - p_v)_m = \frac{\rho_m}{\rho} \frac{\ell_m}{\ell} (p_r - p_v) \qquad (2)$$

For water (from Table B.2):

@ 70°C $\rho_m = 977.8 \text{ kg/m}^3$; $p_{vm} = 3.116 \times 10^4 \text{ N/m}^2$ (abs)

@ 30°C $\rho = 995.7 \text{ kg/m}^3$; $p_v = 4.243 \times 10^3 \text{ N/m}^2$ (abs)

Thus, from Eq. (2)

$$p_{rm} = \left(\frac{977.8 \frac{\text{kg}}{\text{m}^3}}{995.7 \frac{\text{kg}}{\text{m}^3}}\right)\left(\frac{1}{5}\right)\left(101 \times 10^3 \frac{\text{N}}{\text{m}^2} - 4.243 \times 10^3 \frac{\text{N}}{\text{m}^2}\right) + 3.116 \times 10^4 \frac{\text{N}}{\text{m}^2}$$

$$= \underline{\underline{50.2 \text{ kPa (abs)}}}$$