

6.4 The three components of velocity in a flow field are given by

$$u = x^2 + y^2 + z^2$$

$$v = xy + yz + z^2$$

$$w = -3xz - z^2/2 + 4$$

(a) Determine the volumetric dilatation rate, and interpret the results. (b) Determine an expression for the rotation vector. Is this an irrotational flow field?

$$(a) \quad \text{Volumetric dilatation rate} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \quad (\text{Eq. 6.9})$$

Thus, for velocity components given

$$\text{volumetric dilatation rate} = 2x + (x+z) + (-3x-z) = \underline{\underline{0}}$$

This result indicates that there is no change in the volume of a fluid element as it moves from one location to another.

(b) From Eqs. 6.12, 6.13, and 6.14 with the velocity components given:

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = \frac{1}{2} (y - 2y) = -\frac{y}{2}$$

$$\omega_x = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) = \frac{1}{2} [0 - (y + 2z)] = -\left(\frac{y}{2} + z\right)$$

$$\omega_y = \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) = \frac{1}{2} [2z - (-3z)] = \frac{5z}{2}$$

$$\text{Thus, } \underline{\underline{\vec{\omega} = -\left(\frac{y}{2} + z\right) \hat{i} + \frac{5z}{2} \hat{j} - \frac{y}{2} \hat{k}}}$$

Since $\vec{\omega}$ is not zero everywhere the flow field is not irrotational. No.