4.8

4.8 The velocity field of a flow is given by $\mathbf{V} = 20y/(x^2 + y^2)^{1/2}\hat{\mathbf{i}} - 20x/(x^2 + y^2)^{1/2}\hat{\mathbf{j}}$ ft/s, where x and y are in feet. Determine the fluid speed at points along the x axis; along the y axis. What is the angle between the velocity vector and the x axis at points $(x, y) = (5, 0)$, $(5, 5)$, and $(0, 5)$?

$$u = \frac{20y}{(x^2 + y^2)^{1/2}} \quad , \quad v = -\frac{20x}{(x^2 + y^2)^{1/2}}$$

Thus, $V = \sqrt{u^2 + v^2}$ or

$$V = \left[\frac{400x^2 + 400y^2}{(x^2 + y^2)}\right]^{1/2} = \underline{\underline{20 \frac{ft}{s} \text{ for any } x, y}}$$

Also,

$$\tan\theta = \frac{v}{u} = \frac{\frac{-20x}{(x^2+y^2)^{1/2}}}{\frac{20y}{(x^2+y^2)^{1/2}}}$$

or

$$\tan\theta = -\frac{x}{y}$$

Thus, for $(x,y) = (5,0)$

$\tan\theta = -\infty$ or $\theta = \underline{\underline{-90^\circ}}$

for $(x,y) = (5,5)$

$\tan\theta = -1$ or $\theta = \underline{\underline{-45^\circ}}$

for $(x,y) = (0,5)$

$\tan\theta = 0$ or $\theta = \underline{\underline{0^\circ}}$