2.86

2.86 Hoover Dam (see **Video 2.4**) is the highest arch-gravity type of dam in the United States. A cross section of the dam is shown in Fig. P2.86(*a*). The walls of the canyon in which the dam is located are sloped, and just upstream of the dam the vertical plane shown in Figure P2.86(*b*) approximately represents the cross section of the water acting on the dam. Use this vertical cross section to estimate the resultant horizontal force of the water on the dam, and show where this force acts.

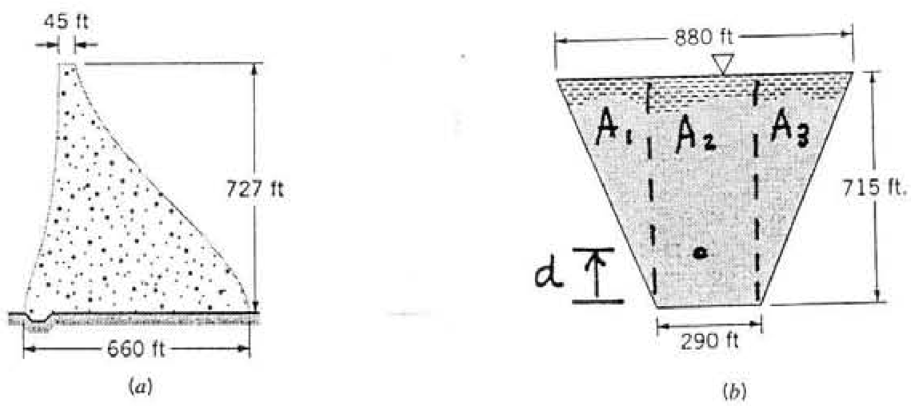

FIGURE P2.86

Break area into 3 parts as shown.

For area 1:

$$F_{R_1} = \gamma h_c A_1 = \left(62.4\ \tfrac{lb}{ft^3}\right)\left(\tfrac{1}{3}\right)(715\ ft)\left(\tfrac{1}{2}\right)(295\ ft)(715\ ft)$$

$$= 1.57 \times 10^9\ lb$$

For area 3: $F_{R_3} = F_{R_1} = 1.57 \times 10^9\ lb$

For area 2:

$$F_{R_2} = \gamma h_c A_2 = \left(62.4\ \tfrac{lb}{ft^3}\right)\left(\tfrac{1}{2}\right)(715\ ft)(290\ ft)(715\ ft)$$

$$= 4.63 \times 10^9\ lb$$

Thus,

$$F_R = F_{R_1} + F_{R_2} + F_{R_3} = 1.57 \times 10^9\ lb + 4.63 \times 10^9\ lb + 1.57 \times 10^9\ lb$$

$$= 7.77 \times 10^9\ lb$$

Since the moment of the resultant force about the base of the dam must be equal to the moments due to F_{R_1}, F_{R_2}, and F_{R_3}, it follows that

(con't)

2.86 (con't)

$$F_R \times d = F_{R_1}\left(\frac{2}{3}\right)(715\text{ ft}) + F_{R_2}\left(\frac{1}{2}\right)(715\text{ ft}) + F_{R_3}\left(\frac{2}{3}\right)(715\text{ ft})$$

and

$$d = \frac{(1.57\times10^9\text{ lb})\left(\frac{2}{3}\right)(715\text{ ft}) + (4.63\times10^9\text{ lb})\left(\frac{1}{2}\right)(715\text{ ft}) + (1.57\times10^9\text{ lb})\left(\frac{2}{3}\right)(715\text{ ft})}{7.77\times10^9\text{ lb}}$$

$$= 406\text{ ft}$$

Thus, the resultant horizontal force on the dam is 7.77×10^9 lb acting 406 ft up from the base of the dam along the axis of symmetry of the area.

2.86 (Con't)

$$F_R \times d = F_{R_1} d_1 + F_{R_2} d_2 + F_{R_3} d_3$$

where,

$$d_1 = 715\ ft - y_{R_1} = d_3$$

$$d_2 = 715\ ft - y_{R_2}$$

$$y_{R_1} = y_{c_1} + \frac{I_{xc_1}}{y_{c_1} A_1} = y_{R_3}$$

$$y_{R_2} = y_{c_2} + \frac{I_{xc_2}}{y_{c_2} A_2}$$

$$y_{c_1} = y_{c_3} = \left(\frac{1}{3}\right)(715\ ft) = 238.3\ ft$$

$$y_{c_2} = \left(\frac{1}{2}\right)(715\ ft) = 357.5\ ft$$

$$I_{xc_1} = \frac{(295\ ft)(715\ ft)^3}{36} = 3.0 \times 10^9\ ft^4$$

$$I_{xc_2} = \frac{(290\ ft)(715\ ft)^3}{12} = 8.8 \times 10^9\ ft^4$$

$$A_1 = \left(\frac{1}{2}\right)(295\ ft)(715\ ft) = 1.1 \times 10^5\ ft^2$$

$$A_2 = (295\ ft)(715\ ft) = 2.1 \times 10^5\ ft^2$$

thus,

$$d_1 = d_3 = 715\ ft - \left(238.3\ ft + \frac{3.0\times10^9\ ft^4}{(238.3\ ft)(1.1\times10^5\ ft^2)}\right) = 362.3\ ft$$

$$d_2 = 715\ ft - \left(357.5\ ft + \frac{8.8\times10^9\ ft^4}{(357.5\ ft)(2.1\times10^5\ ft^2)}\right) = 240.3\ ft$$

$$\therefore d = \frac{(1.57 \times 10^9\ lb)(362.3\ ft) + (4.63 \times 10^9\ lb)(240.3\ ft) + (1.57 \times 10^9\ lb)(362.3\ ft)}{7.77 \times 10^9\ lb}$$

$$= 289.6\ ft$$

Thus, the resultant horizontal force on the dam is 7.77×10^9 lb acting 289.6 ft up from the base of the dam along the axis of symmetry of the area.