7.22 The pressure drop, Δp, along a straight pipe of diameter D has been experimentally studied, and it is observed that for laminar flow of a given fluid and pipe, the pressure drop varies directly with the distance, ℓ, between pressure taps. Assume that Δp is a function of D and ℓ, the velocity, V, and the fluid viscosity, μ. Use dimensional analysis to deduce how the pressure drop varies with pipe diameter.

$$\Delta p = f(D, \ell, V, \mu)$$

$$\Delta p \doteq FL^{-2} \quad D \doteq L \quad \ell \doteq L \quad V \doteq LT^{-1} \quad \mu \doteq FL^{-2}T$$

From the pi theorem, $5 - 3 = 2$ pi terms required.

By inspection, for π_1 (containing Δp):

$$\pi_1 = \frac{\Delta p\, D}{\mu V} \doteq \frac{(FL^{-2})(L)}{(FL^{-2}T)(LT^{-1})} \doteq F^0 L^0 T^0$$

Check using MLT:

$$\frac{\Delta p\, D}{\mu V} \doteq \frac{(ML^{-1}T^{-2})(L)}{(ML^{-1}T^{-1})(LT^{-1})} \doteq M^0 L^0 T^0 \quad \therefore ok$$

For π_2 (containing ℓ):

$$\pi_2 = \frac{\ell}{D}$$

which is obviously dimensionless. Thus,

$$\frac{\Delta p\, D}{\mu V} = \phi\left(\frac{\ell}{D}\right) \tag{1}$$

From the statement of the problem, $\Delta p \propto \ell$ so that Eq. (1) must be of the form

$$\frac{\Delta p\, D}{\mu V} = K \frac{\ell}{D}$$

where K is some constant. It thus follows that

$$\underline{\underline{\Delta p \propto \frac{1}{D^2}}}$$

for a given velocity.