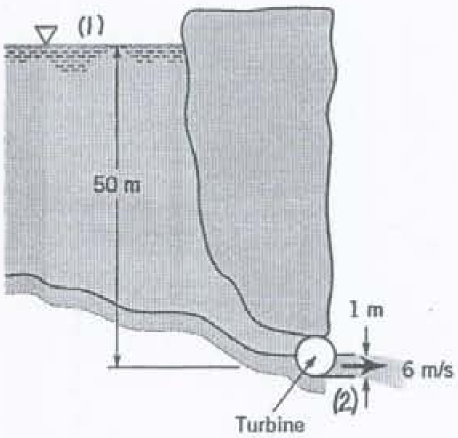
5.108

**5.108** What is the maximum possible power output of the hydroelectric turbine shown in Fig.P5.108?




■ FIGURE P5.108

For flow from section (1) to section (2), Eq. 5.82 yields

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 + w_{\substack{shaft \\ net\ in}} - loss \qquad (1)$$

Since $p_1 = p_2 = p_{atm}$, $w_{\substack{shaft \\ net\ in}} = -w_{\substack{shaft \\ net\ out}}$ Eq. 1 can be expressed as

$$w_{\substack{shaft \\ net\ out}} = g(z_1 - z_2) - \frac{V_2^2}{2} - loss$$

The maximum work or power output is achieved when $loss = 0$.

Thus

$$\dot{W}_{\substack{shaft \\ net\ out \\ maximum}} = \dot{m}\, w_{\substack{shaft \\ net\ out \\ maximum}} = \dot{m}\left[g(z_1 - z_2) - \frac{V_2^2}{2}\right]$$

Now

$$\dot{m} = \rho V_2 A_2 = \rho V_2 \frac{\pi D_2^2}{4} = \left(999 \frac{kg}{m^3}\right)\left(6 \frac{m}{s}\right)\frac{\pi (1\,m)^2}{4} = 4710 \frac{kg}{s}$$

and

$$\dot{W}_{\substack{shaft \\ net\ out \\ maximum}} = \left(4710 \frac{kg}{s}\right)\left[\left(9.81 \frac{m}{s^2}\right)(50\,m) - \frac{\left(6 \frac{m}{s}\right)^2}{2}\right]\left(\frac{1\,N}{kg \cdot \frac{m}{s^2}}\right)$$

$$\dot{W}_{\substack{shaft \\ net\ out \\ maximum}} = \underline{\underline{2.22 \times 10^6 \frac{N \cdot m}{s}}} = \underline{\underline{2.22 \times 10^6\, W}} = \underline{\underline{2.22}}\ MW$$