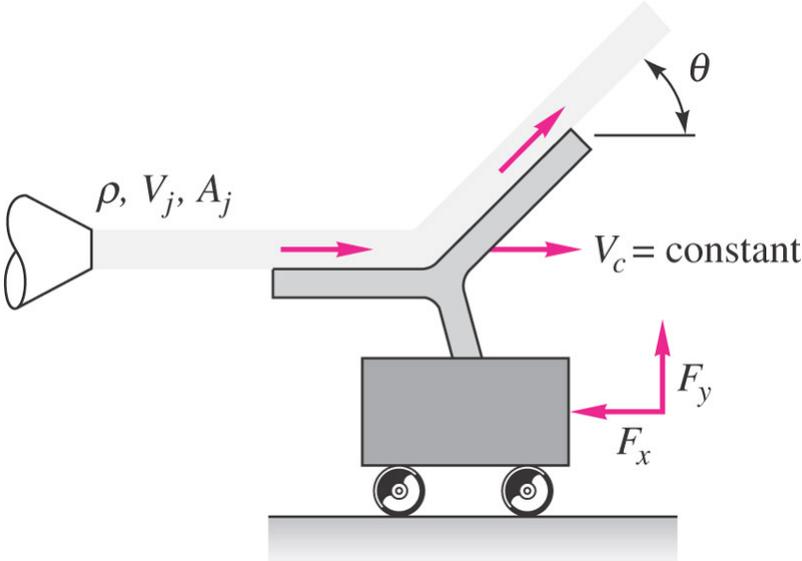


Ex) A jet strikes a vane which moves to the right at constant velocity V_C on a frictionless cart. Compute (a) the force F_x required to restrain the cart and (b) the power P delivered to the cart. Also find the cart velocity for which (c) the force F_x is a maximum and (d) the power P is a maximum.



Solution:

$$\underline{\Sigma F} = \frac{d}{dt} \int_{CV} \rho \underline{V} dV + \int_{CS} \rho \underline{V} \underline{V}_R \cdot \underline{n} dA$$

where, $\underline{V}_R = \underline{V} - \underline{V}_{CS}$

Assume relative inertial coordinates with non-deforming CV i.e. CV moves at constant translational non-accelerating

$$\underline{V}_{CS} = u_{CS} \hat{i} + v_{CS} \hat{j} + w_{CS} \hat{k} = V_C \hat{i}$$

and steady flow $\underline{V} \neq \underline{V}(t)$ with $\rho = \text{constant}$

Continuity:

$$0 = \frac{d}{dt} \int_{CV} \rho dV + \int_{CS} \rho \underline{V}_R \cdot \underline{n} dA$$

$$0 = \rho \int_{CS} \underline{V}_R \cdot \underline{n} dA$$

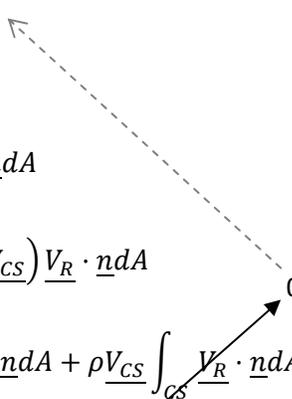
Momentum:

$$\underline{\Sigma F} = \int_{CS} \rho \underline{V} \underline{V}_R \cdot \underline{n} dA$$

$$= \int_{CS} \rho (\underline{V}_R + \underline{V}_{CS}) \underline{V}_R \cdot \underline{n} dA$$

$$= \rho \int_{CS} \underline{V}_R \underline{V}_R \cdot \underline{n} dA + \rho \underline{V}_{CS} \int_{CS} \underline{V}_R \cdot \underline{n} dA$$

$$\underline{\Sigma F} = \rho \int_{CS} \underline{V}_R \underline{V}_R \cdot \underline{n} dA$$



$$\sum F_x = -F_x = \rho \int_{CS} V_{Rx} V_R \cdot \underline{n} dA$$

$$-F_x = \rho V_{R_{x1}} (-V_{R1} A_1) + \rho V_{R_{x2}} (V_{R2} A_2)$$

$$0 = \rho \int_{CS} V_R \cdot \underline{n} dA$$

$$-\rho V_{R1} A_1 + \rho V_{R2} A_2 = 0$$

$$V_{R1} A_1 = V_{R2} A_2 = \frac{(V_j - V_C)}{V_{R1} = V_{R_{x1}} = V_j - V_C} A_j$$

$$-F_x = \rho (V_j - V_C) [-(V_j - V_C) A_j] + \rho (V_j - V_C) \cos \theta (V_j - V_C) A_j$$

$$F_x = \rho (V_j - V_C)^2 A_j [1 - \cos \theta]$$

$$Power = V_C F_x = V_C \rho (V_j - V_C)^2 A_j (1 - \cos \theta)$$

$$F_{x_{max}} = \rho V_j^2 A_j (1 - \cos \theta), \quad V_C = 0$$

$$P_{max} \Rightarrow \frac{dP}{dV_C} = 0$$

$$P = V_C \rho (V_j^2 - 2V_C V_j + V_C^2) A_j (1 - \cos \theta)$$

$$= \rho (V_j^2 V_C - 2V_C^2 V_j + V_C^3) A_j (1 - \cos \theta)$$

$$\frac{dP}{dV_C} = \rho (V_j^2 - 4V_C V_j + 3V_C^2) A_j (1 - \cos \theta) = 0$$

$$3V_C^2 - 4V_j V_C + V_j^2 = 0$$

$$V_C = \frac{+4V_j \pm \sqrt{16V_j^2 - 12V_j^2}}{6} = \frac{4V_j \pm 2V_j}{6}$$

$$V_C = \frac{V_j}{3}$$

$$\begin{aligned} P_{max} &= \frac{V_j}{3} \rho \left(\frac{2V_j}{3} \right)^2 A_j (1 - \cos \theta) \\ &= \frac{4}{27} V_j^3 \rho A_j (1 - \cos \theta) \end{aligned}$$