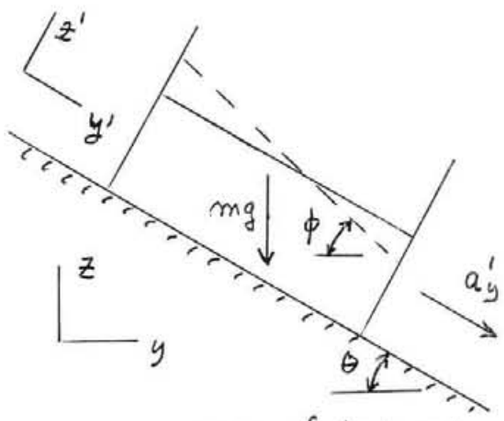
2.114

2.114 If the tank of Problem 2.113 slides down a frictionless plane that is inclined at 30° with the horizontal, determine the angle the free surface makes with the horizontal.



$m \sim$ mass of tank and gasoline

From Newton's 2nd law,

$$\Sigma F_{y'} = m a_{y'}$$

Since the only force in the y'-direction is the component of weight $(mg)\sin\theta$,

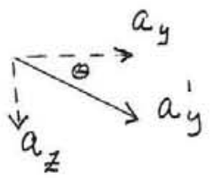
$$(mg)\sin\theta = m a_{y'}$$

so that

$$a_{y'} = g \sin\theta$$

and therefore

$$a_y = a_{y'} \cos\theta \qquad a_z = -a_{y'} \sin\theta$$



Also,

$$\frac{dz}{dy} = -\frac{a_y}{g + a_z} \qquad \text{(Eq. 2.28)}$$

$$= -\frac{a_{y'} \cos\theta}{g - a_{y'} \sin\theta} = -\frac{g \sin\theta \cos\theta}{g - g \sin\theta \sin\theta}$$

$$= -\frac{\sin\theta \cos\theta}{1 - \sin^2\theta} = -\frac{\sin\theta \cos\theta}{\cos^2\theta} = -\tan\theta$$

Hence, $\frac{dz}{dy} = -\tan\theta$, so that the free surface is at the same angle as the plane.