6.23

6.23 In a two-dimensional, incompressible flow field, the x component of velocity is given by the equation $u = 2x$. **(a)** Determine the corresponding equation for the y component of velocity if $v = 0$ along the x axis. **(b)** For this flow field what is the magnitude of the average velocity of the fluid crossing the surface OA of Fig. P6.23? Assume that the velocities are in ft/s when x and y are in feet.

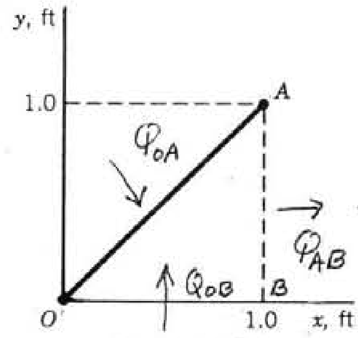

FIGURE P6.23

(a) To satisfy the continuity equation (consider a unit thickness = 1 ft)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Since $\frac{\partial u}{\partial x} = 2$

it follows that

$$\frac{\partial v}{\partial y} = -2 \qquad (1)$$

Integration of Eq. (1) with respect to y yields

$$v = -2y + f(x)$$

If $v = 0$ along x-axis ($y=0$) then $f(x) = 0$ so that

$$\underline{\underline{v = -2y}}$$

(b) To satisfy conservation of mass

$$Q_{OA} = Q_{AB} - Q_{OB} \quad \text{(see figure)}$$

Along AB $\quad u = 2(1) = 2\ \frac{ft}{s}$ so that

$$Q_{AB} = u\,A_{AB} = (2\ ft/s)(1\ ft)(1\ ft) = 2\ \frac{ft^3}{s}$$

Along OB $\quad v = 0$ so that $Q_{OB} = 0$.

Thus,

$$Q_{OA} = Q_{AB} = 2\ \frac{ft^3}{s}$$

and

$$V_{AV} = \frac{Q_{OA}}{area_{OA}} = \frac{2\ \frac{ft^3}{s}}{\sqrt{2}\ ft^2} = \underline{\underline{1.41\ \frac{ft}{s}}}$$