3.75

3.75 Water flows from a large tank as shown in Fig. P3.75. Atmospheric pressure is 14.5 psia and the vapor pressure is 1.60 psia. If viscous effects are neglected, at what height, h, will cavitation begin? To avoid cavitation, should the value of D_1 be increased or decreased? To avoid cavitation, should the value of D_2 be increased or decreased? Explain.

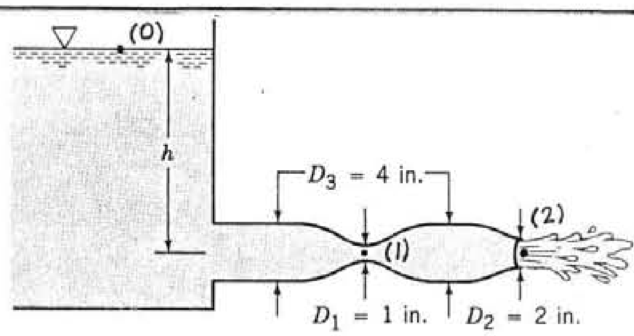

FIGURE P3.75

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1$$ where $p_0 = 14.5\,psia$, $p_1 = 1.60\,psia$, $z_0 = h$, $z_1 = 0$, and $V_0 = 0$

Thus,

$$h = \frac{p_1 - p_0}{\gamma} + \frac{V_1^2}{2g} \qquad (1)$$

However,

$A_1 V_1 = A_2 V_2$ or $V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2$

where

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ with $p_0 = p_2$ and $z_2 = 0$

Thus,

$$\frac{V_2^2}{2g} = h$$

so that

$$\frac{V_1^2}{2g} = \frac{\left(\frac{D_2}{D_1}\right)^4 V_2^2}{2g} = \left(\frac{D_2}{D_1}\right)^4 h \qquad (2)$$

Combine Eqs. (1) and (2) to obtain

$$h = \frac{p_1 - p_0}{\gamma} + \left(\frac{D_2}{D_1}\right)^4 h$$

or

$$h = \frac{p_0 - p_1}{\gamma\left[\left(\frac{D_2}{D_1}\right)^4 - 1\right]} = \frac{(14.5 - 1.60)\frac{lb}{in.^2}\left(144\frac{in.^2}{ft^2}\right)}{62.4\frac{lb}{ft^3}\left[\left(\frac{2\,in.}{1\,in.}\right)^4 - 1\right]} = \underline{\underline{1.98\,ft}} \qquad (3)$$

From Eq. (3) it is seen that h increases in increasing D_1 and decreasing D_2. Thus, to avoid cavitation (i.e. to have h small enough) $\underline{\underline{D_1 \text{ should be increased and } D_2 \text{ decreased.}}}$