2.88

2.88 The homogeneous gate shown in Fig. P2.88 consists of one quarter of a circular cylinder and is used to maintain a water depth of 4 m. That is, when the water depth exceeds 4 m, the gate opens slightly and lets the water flow under it. Determine the weight of the gate per meter of length.

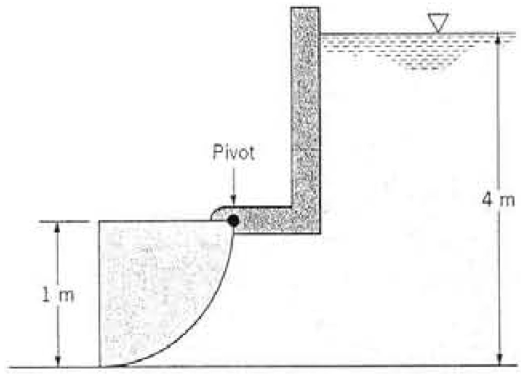
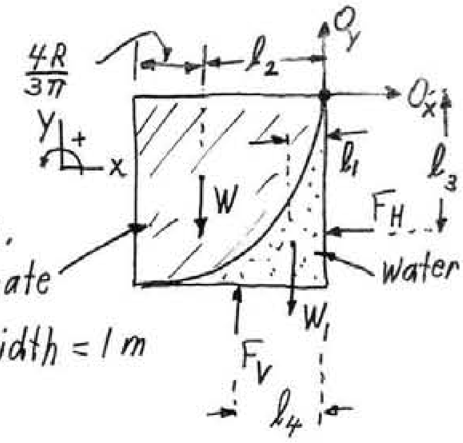


FIGURE P2.88



Consider the free body diagram of the gate and a portion of the water as shown.

$\Sigma M_O = 0$, or

(1) $\ell_2 W + \ell_1 W_1 - F_H \ell_3 - F_V \ell_4 = 0$, where

(2) $F_H = \gamma h_c A = 9.8\times 10^3 \frac{N}{m^3}(3.5\,m)(1\,m)(1\,m) = 34.3\,kN$

since for the vertical side, $h_c = 4m - 0.5m = 3.5m$

Also,

(3) $F_V = \gamma h_c A = 9.8\times 10^3 \frac{N}{m^3}(4m)(1m)(1m) = 39.2\,kN$

Also,

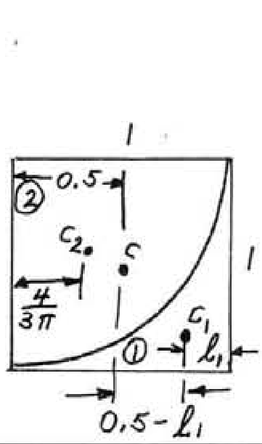
(4) $W_1 = \gamma(1m)^3 - \gamma\left(\frac{\pi}{4}(1m)^2\right)(1m) = 9.8\times 10^3 \frac{N}{m^3}\left[1-\frac{\pi}{4}\right]m^3 = 2.10\,kN$

(5) Now, $\ell_4 = 0.5\,m$ and

(6) $\ell_3 = 0.5\,m + (y_R - y_c) = 0.5\,m + \frac{I_{xc}}{y_c A} = 0.5\,m + \frac{\frac{1}{12}(1m)(1m)^3}{3.5\,m\,(1m)(1m)} = 0.524\,m$

(7) and $\ell_2 = 1m - \frac{4R}{3\pi} = 1 - \frac{4(1m)}{3\pi} = 0.576\,m$

To determine ℓ_1, consider a unit square that consits of a quarter circle and the remainder as shown in the figure. The centroids of areas ① and ② are as indicated.



Thus,

$(0.5 - \frac{4}{3\pi})A_2 = (0.5 - \ell_1)A_1$

(con't)

2.88 (con't)

so that with $A_2 = \frac{\pi}{4}(1)^2 = \frac{\pi}{4}$ and $A_1 = 1 - \frac{\pi}{4}$ this gives

$$\left(0.5 - \frac{4}{3\pi}\right)\frac{\pi}{4} = (0.5 - \ell_1)\left(1 - \frac{\pi}{4}\right)$$

or

(8) $\ell_1 = 0.223\ m$

Hence, by combining Eqs (1) through (8):

$$(0.576\ m)W + (0.223\ m)(2.10\ kN) - (34.3\ kN)(0.524\ m) - (39.2\ kN)(0.5\ m) = 0$$

or

$W = \underline{\underline{64.4\ kN}}$