5.64

5.64 A Pelton wheel vane directs a horizontal, circular cross-sectional jet of water symmetrically as indicated in Fig. P5.64 and Video V5.6. The jet leaves the nozzle with a velocity of 100 ft/s. Determine the x direction component of anchoring force required to (a) hold the vane stationary, (b) confine the speed of the vane to a value of 10 ft/s to the right. The fluid speed magnitude remains constant along the vane surface.

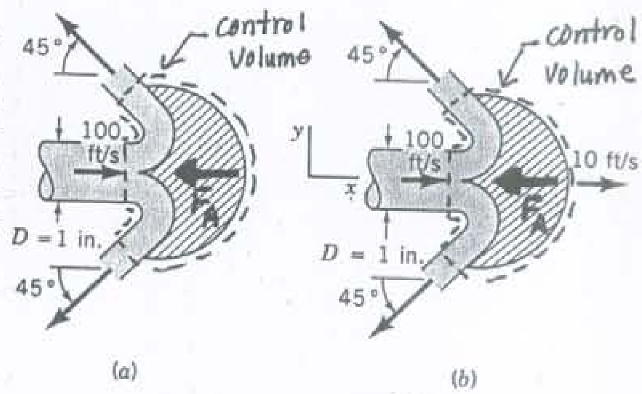

FIGURE P5.64

(a) To determine the x-direction component of anchoring force required to hold the vane stationary we use the stationary control volume shown above and the x-direction component of the linear momentum equation (Eq. 5.22). Thus,

$$F_A = \dot{m}(V_1 + V_2 \cos 45^\circ) = \rho A_1 V_1 (V_1 + V_2 \cos 45^\circ) = \rho \frac{\pi D_1^2}{4} V_1 (V_1 + V_2 \cos 45^\circ)$$

or

$$F_A = \left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right) \frac{\pi (1 \text{ in.})^2 \left(100 \frac{\text{ft}}{\text{s}}\right)}{(4)\left(12 \frac{\text{in.}}{\text{ft}}\right)^2} \left[\left(100 \frac{\text{ft}}{\text{s}}\right) + \left(100 \frac{\text{ft}}{\text{s}}\right) \cos 45^\circ\right] \left(\frac{1 \text{ lb}}{\text{slug} \cdot \frac{\text{ft}}{\text{s}^2}}\right)$$

and

$$F_A = \underline{\underline{181 \text{ lb}}}$$

(b) To determine the x-direction component of anchoring force required to confine the vane to a constant speed of $10 \frac{\text{ft}}{\text{s}}$ to the right we use a control volume moving to the right with a speed of $10 \frac{\text{ft}}{\text{s}}$ and the x-direction component of the linear momentum equation for a translating control volume (Eq. 5.29). Thus,

$$F_A = \rho A_1 W_1 (W_1 + W_2 \cos 45^\circ) = \rho \frac{\pi D_1^2}{4} W_1 (W_1 + W_2 \cos 45^\circ) \qquad (1)$$

We note that

$$W_1 = V_1 - 10 \frac{\text{ft}}{\text{s}} = 100 \frac{\text{ft}}{\text{s}} - 10 \frac{\text{ft}}{\text{s}} = 90 \frac{\text{ft}}{\text{s}}$$

Thus, Eq. 1 leads to

$$F_A = \left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right) \frac{\pi (1 \text{ in.})^2}{4 \left(12 \frac{\text{in.}}{\text{ft}}\right)^2} \left(90 \frac{\text{ft}}{\text{s}}\right) \left[90 \frac{\text{ft}}{\text{s}} + \left(90 \frac{\text{ft}}{\text{s}}\right) \cos 45^\circ\right] \left(\frac{1 \text{ lb}}{\text{slug} \cdot \frac{\text{ft}}{\text{s}^2}}\right)$$

or

$$F_A = \underline{\underline{146 \text{ lb}}}$$