4.37

4.37 Repeat Problem 4.36 with the assumption that the flow is not steady, but at the time when $V_1 = 10$ m/s and $V_2 = 25$ m/s, it is known that $\partial V_1/\partial t = 20$ m/s^2 and $\partial V_2/\partial t = 60$ m/s^2.

With $u = u(x,t)$, $v = 0$, and $w = 0$ the acceleration $\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ can be written as

$$\vec{a} = a_x \hat{\imath} \text{ where } a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}, \text{ with } u = a(t)x + b(t). \qquad (1)$$

At the given time ($t = t_0$) $u = V_1 = 10\,\frac{m}{s}$ at $x = 0$ and $u = V_2 = 25\,\frac{m}{s}$ at $x = 1m$

Thus, $10 = 0 + b(t_0)$

$25 = a(t_0) + b(t_0)$ so that $a(t_0) = 15$ and $b(t_0) = 10$

Also at $t = t_0$, $\frac{\partial u}{\partial t} = \frac{\partial V_1}{\partial t} = \underline{\underline{20\,\frac{m}{s^2}}}$ at $x = 0$

and $\frac{\partial u}{\partial t} = \frac{\partial V_2}{\partial t} = \underline{\underline{60\,\frac{m}{s^2}}}$ at $x = 1m$ Note: These are local accelerations at time $t = t_0$

The convective acceleration at $x = 0$ (Eq. (1)) is

$$u\frac{\partial u}{\partial x} = (ax + b)(a) = (15(0) + 10)\frac{m}{s}(15\,\frac{1}{s}) = \underline{\underline{150\,\frac{m}{s^2}}}$$

while at $x = 1$ it is

$$u\frac{\partial u}{\partial x} = (15(1) + 10)\frac{m}{s}(15\,\frac{1}{s}) = \underline{\underline{375\,\frac{m}{s^2}}}$$

The fluid acceleration at $t = t_0$ is

$$\vec{a} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{\imath} = (20 + 150)\hat{\imath}\,\frac{m}{s^2} = \underline{\underline{170\hat{\imath}\,\frac{m}{s^2} \text{ at } x = 0}}$$

and

$$\vec{a} = (60 + 375)\hat{\imath}\,\frac{m}{s^2} = \underline{\underline{435\hat{\imath}\,\frac{m}{s^2} \text{ at } x = 1m}}$$