4.33

4.33 A fluid flows along the x axis with a velocity given by $\mathbf{V} = (x/t)\hat{\mathbf{i}}$, where x is in feet and t in seconds. **(a)** Plot the speed for $0 \le x \le 10$ ft and $t = 3$ s. **(b)** Plot the speed for $x = 7$ ft and $2 \le t \le 4$ s. **(c)** Determine the local and convective acceleration. **(d)** Show that the acceleration of any fluid particle in the flow is zero. **(e)** Explain physically how the velocity of a particle in this unsteady flow remains constant throughout its motion.

(a) $u = \frac{x}{t}\ \frac{ft}{s}$ so at $t = 3$ s, $u = \frac{x}{3}\ \frac{ft}{s}$

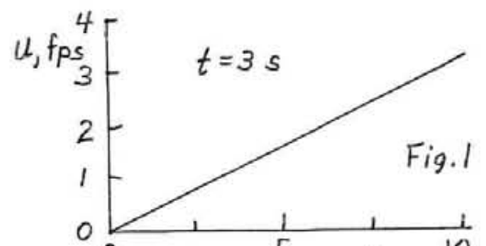

Fig. 1

(b) For $x = 7$ ft, $u = \frac{7}{t}\ \frac{ft}{s}$

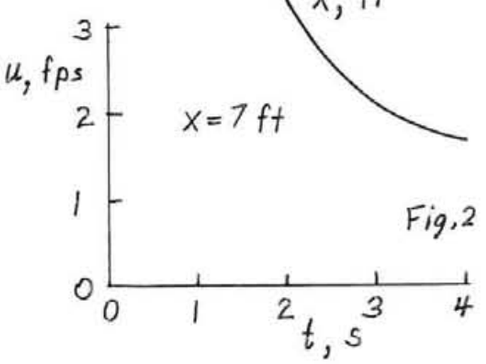

Fig. 2

(c) $\frac{\partial u}{\partial t} = -\frac{x}{t^2}$ and $u\frac{\partial u}{\partial x} = \frac{x}{t}\left(\frac{1}{t}\right) = \frac{x}{t^2}$

(d) For any fluid particle $\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ which with $v = 0$, $w = 0$ becomes

$$\vec{a} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{i} = \left(-\frac{x}{t^2} + \frac{x}{t^2}\right)\hat{i} \equiv 0$$

(e) The particles flow into areas of higher velocity (see Fig. 1), but at any given location the velocity is decreasing in time (see Fig. 2). For the given velocity field the local and convective accelerations are equal and opposite, giving zero acceleration throughout.