4.5

4.5 A two-dimensional velocity field is given by $u = 1 + y$ and $v = 1$. Determine the equation of the streamline that passes through the origin. On a graph, plot this streamline.

$u = 1+y$ and $v = 1$ so the streamlines are given by

$$\frac{dy}{dx} = \frac{v}{u} = \frac{1}{1+y}$$

Thus,

$$\int (1+y)\,dy = \int dx \quad \text{or}$$

$y + \frac{1}{2}y^2 = x + C$, where C is a constant.

For the streamline that goes through $x=y=0$, $C=0$.

Hence,

$$x = y + \tfrac{1}{2}y^2$$

This streamline is plotted below. Note that since $v = 1 > 0$, the direction of flow is as shown.