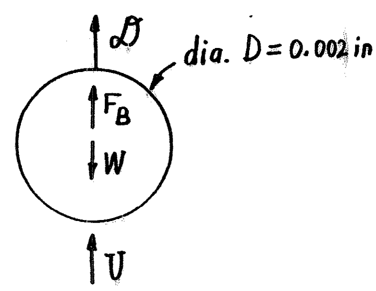
9.41

9.41 A small spherical water drop of diameter 0.002 in. exists in the atmosphere at 5000-ft altitude. Will the drop rise or fall if it is in a thermal (an upward flowing column of air) having a speed of 4 ft/s? Repeat for speeds of 1 ft/s and 0.1 ft/s.



In stationary air the particle falls with speed U such that $\mathcal{D} + F_B = W$, where if $Re = \frac{UD}{\nu} < 1$ then

$\mathcal{D} = \text{drag} = 3\pi D U \mu$ Also, $W = \gamma_{H_2O} \forall = \gamma_{H_2O} \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 = \text{weight}$ (1)

and $F_B = \gamma_{air} \forall = \gamma_{air} \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 = \text{buoyant force}$

Since $\gamma_{air} \ll \gamma_{H_2O}$ we can neglect the buoyant force.

That is, $\mathcal{D} = W$, or

$3\pi D U \mu = \gamma_{H_2O} \frac{4\pi}{3}\left(\frac{D}{2}\right)^3$ or $U = \frac{\gamma_{H_2O} D^2}{18 \mu}$

At an altitude of 5000 ft, $\mu = 3.637 \times 10^{-7} \frac{lb \cdot s}{ft^2}$

so that

$$U = \frac{(62.4 \frac{lb}{ft^3})\left(\frac{0.002}{12} ft\right)^2}{18\left(3.637 \times 10^{-7} \frac{lb \cdot s}{ft^2}\right)} = 0.265 \frac{ft}{s}$$

Thus, the drop will rise if the upward velocity is $4 \frac{ft}{s}$ or $1 \frac{ft}{s}$, but it will fall if it is $0.1 \frac{ft}{s}$.

Note: The above is correct if $Re < 1$. Since $Re = \frac{\rho U D}{\mu}$

or $Re = \frac{(2.048 \times 10^{-3} \frac{slug}{ft^3})(0.265 \frac{ft}{s})(\frac{0.002}{12} ft)}{3.637 \times 10^{-7} \frac{lb \cdot s}{ft^2}} = 0.249$ the low Re drag equation, Eq. (1), is valid.