9.15

9.15 Air enters a square duct through a 1-ft opening as is shown in Fig. P9.15. Because the boundary layer displacement thickness increases in the direction of flow, it is necessary to increase the cross-sectional size of the duct if a constant $U = 2$ ft/s velocity is to be maintained outside the boundary layer. Plot a graph of the duct size, d, as a function of x for $0 \le x \le 10$ ft if U is to remain constant. Assume laminar flow.

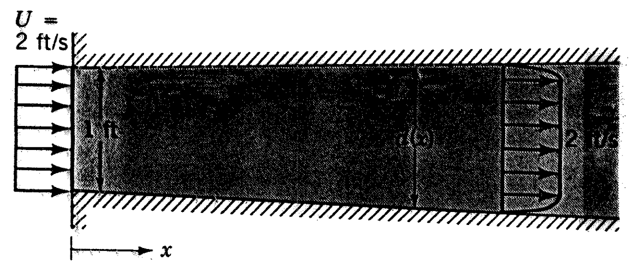

FIGURE P9.15

For incompressible flow $Q_0 = Q(x)$ where Q_0 = flowrate into the duct $= UA_0 = (2\,\frac{ft}{s})(1\,ft^2) = 2\,\frac{ft^3}{s}$ and

$Q(x) = UA$, where $A = (d - 2\delta^*)^2$ is the effective area of the duct (allowing for the decreased flowrate in the boundary layer).

Thus,

$$Q_0 = U(d - 2\delta^*)^2 \quad \text{or} \quad d = 1\,ft + 2\delta^*, \qquad (1)$$

where

$$\delta^* = 1.721\sqrt{\frac{\nu x}{U}} = 1.721\left[\frac{(1.57\times10^{-4}\,\frac{ft^2}{s})\,x}{2\,\frac{ft}{s}}\right]^{1/2} = 0.0152\sqrt{x}\ ft, \text{ where } x \sim ft$$

Hence, from Eq. (1)

$$\underline{\underline{d = 1 + 0.0304\sqrt{x}\ ft}}$$

For example, $d = 1$ ft at $x = 0$ and $d = 1.096$ ft at $x = 10$ ft.