9.14

9.14 Because of the velocity deficit, $U - u$, in the boundary layer, the streamlines for flow past a flat plate are not exactly parallel to the plate. This deviation can be determined by use of the displacement thickness, δ^*. For air blowing past the flat plate shown in Fig. P9.14, plot the streamline A–B that passes through the edge of the boundary layer ($y = \delta_B$ at $x = \ell$) at point B. That is, plot $y = y(x)$ for streamline A–B. Assume laminar boundary layer flow.

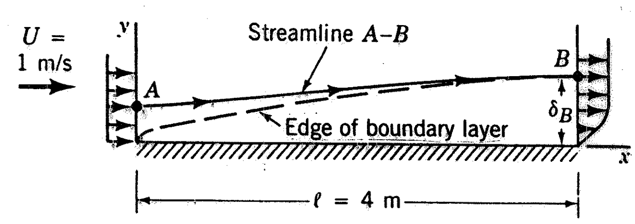
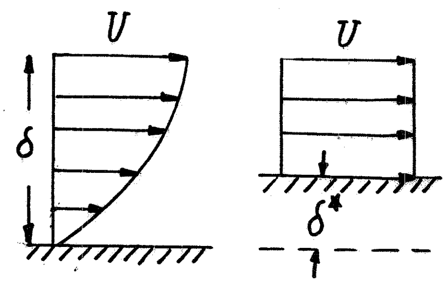


FIGURE P9.14

Since $Re_\ell = \frac{U\ell}{\nu} = \frac{(1\,\frac{m}{s})(4m)}{1.46\times 10^{-5}\,\frac{m^2}{s}} = 2.74\times 10^5 < 5\times 10^5$, the boundary layer flow remains laminar along the entire plate. Hence,

$$\delta = 5\sqrt{\frac{\nu x}{U}} \quad \text{or} \quad \delta_B = 5\left[\frac{(1.46\times10^{-5}\,\frac{m^2}{s})(4m)}{1\,\frac{m}{s}}\right]^{1/2} = 0.0382\,m$$

The flowrate carried by the actual boundary layer is by definition equal to that carried by a uniform velocity with the plate displaced by an amount δ^*. Since there is no flow through the plate or streamline A-B,



$$Q_A = Q_B, \text{ or } U y_A = (\delta_B - \delta_B^*)U \qquad (1)$$

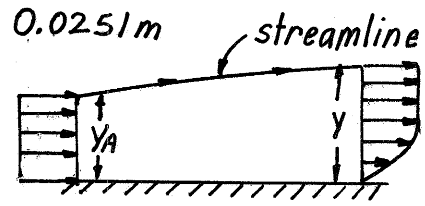
where $\delta^* = 1.721\sqrt{\frac{\nu x}{U}}$

or $\delta_B^* = 1.721\left[\frac{(1.46\times10^{-5}\,\frac{m^2}{s})(4m)}{1\,\frac{m}{s}}\right]^{1/2} = 0.01315\,m$

Thus,

$$y_A = \delta_B - \delta_B^* = 0.0382\,m - 0.01315\,m = 0.0251\,m$$

Hence, for any x-location



$Q_A = Q$ or $U y_A = U(y - \delta^*)$

or $y = y_A + \delta^* = y_A + 1.721\sqrt{\frac{\nu x}{U}}$

$$= 0.0251\,m + 1.721\left[\frac{(1.46\times10^{-5}\,\frac{m^2}{s})\,x\,m}{1\,\frac{m}{s}}\right]^{\frac{1}{2}} = \underline{\underline{0.0251 + 6.58\times10^{-3}\sqrt{x}\ m,}}$$

where $x \sim m$