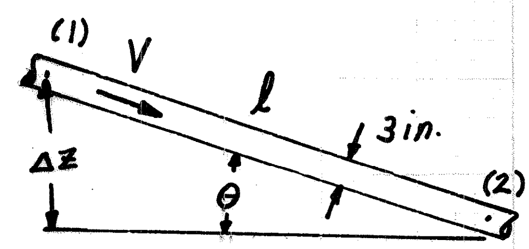
8.12

8.12 Water flows downhill through a 3-in.-diameter steel pipe. The slope of the hill is such that for each mile (5280 ft) of horizontal distance, the change in elevation is Δz ft. Determine the maximum value of Δz if the flow is to remain laminar and the pressure all along the pipe is constant.



For laminar flow $V = \dfrac{(\Delta p - \gamma \ell \sin\theta) D^2}{32 \mu \ell}$ (1)

where for this case

$\Delta p = 0$, $\ell = 5280\ ft$, $D = 0.25\ ft$ and for maximum Δz, $Re = 2100$

Thus,

$\dfrac{\rho V D}{\mu} = 2100$ or $V = \dfrac{2100 \mu}{\rho D} = \dfrac{2100\left(2.34\times10^{-5} \frac{lb\cdot s}{ft^2}\right)}{1.94 \frac{slug}{ft^3}(0.25 ft)} = 0.101 \frac{ft}{s}$

Hence, from Eq. (1):

$\sin\theta = -\dfrac{32 \mu V}{\gamma D^2} = -\dfrac{32\left(2.34\times10^{-5} \frac{lb\cdot s}{ft^2}\right)\left(0.101 \frac{ft}{s}\right)}{62.4 \frac{lb}{ft^3}(0.25 ft)^2} = -1.94\times10^{-5}$

or

$\Delta z = -\ell \sin\theta = -(5280 ft) \times (-1.94\times10^{-5}) = \underline{\underline{0.102\ ft}}$

Note: Could use the energy equation

$z_1 - f \dfrac{\ell}{D} \dfrac{V^2}{2g} = z_2$ with $Re = 2100$ so that $V = 0.101 \frac{ft}{s}$ and

$f = \dfrac{64}{Re} = \dfrac{64}{2100} = 0.0305$

and obtain the same result: $z_1 - z_2 = 0.102\ ft$